Math 3333 Name:

HW 8

Please, write clearly and justify all your statements using the material covered in class to get credit for your work.

(1) Let $f : D \to \mathbb{R}$ be continuous at $c \in D$. Prove that there exists an $M > 0$ and a neighborhood U of c such that $|f(x)| \leq M$ for all $x \in U \cap D$.

(2) Determine the following limit

$$\lim_{x \to 0-} \frac{4x}{|x|}$$

(a) using the sequential definition;
(b) using the $\epsilon - \delta$ definition.

(3) Let $f : \mathbb{R} \to \mathbb{R}$ be given by

$$f(x) = \begin{cases} \sin(1/x) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0. \end{cases}$$

(a) Show that f is not continuous at $x = 0$.
(b) Show that f has the intermediate property on any interval $[a, b] \in \mathbb{R}$, that is, if k is any value between $f(a)$ and $f(b)$, then there exists $c \in (a, b)$ such that $f(c) = k$.

(4) Show that any polynomial p of odd degree has at least one real root.

(5) Let $f : [a, b] \to [a, b]$ be continuous. Prove that f must have a fixed point, that is, there is $c \in [a, b]$ such that $f(c) = c$. [Hint: Set $h(x) = f(x) - x$ and apply the Intermediate Value Theorem.]