

HW 8

Please, write clearly and justify all your statements using the material covered in class to get credit for your work.

(1) Prove that the function $f(x) = \frac{1}{x}$ on $[2, \infty)$ is uniformly continuous by verifying the $\epsilon - \delta$ property.

(2) Let $f : D \rightarrow \mathbb{R}$ be continuous at $c \in D$. Prove that there exists an $M > 0$ and a neighborhood U of c such that $|f(x)| \leq M$ for all $x \in U \cap D$.

(3) Prove that $f(x) = \sqrt{x}$ is uniformly continuous on $[0, \infty)$.

(4) Let f and g be two real-valued functions that are uniformly continuous on a set D . Prove that $f + g$ is uniformly continuous on D .

(5) Find two real-valued functions f and g that are uniformly continuous on a set D , but such that their product $f g$ is not uniformly continuous on D .

(6) Suppose that $f : \mathbb{R} \rightarrow \mathbb{R}$ is continuous and periodic. Prove that f is bounded and uniformly continuous on \mathbb{R} .

(7) Determine the following limit

$$\lim_{x \rightarrow 0^-} \frac{4x}{|x|}$$

(a) using the sequential definition;

(b) using the $\epsilon - \delta$ definition.