Math 3333 Name:

HW 9

(1) Suppose that $f:(a,b)\to\mathbb{R}$ is continuous and that $f(r)=0$ for every rational number $r\in(a,b)$. Prove that $f(x)=0$ for all $x\in(a,b)$.

(2) Let $f:D\to\mathbb{R}$ and $c\in D$. We say that f is *bounded on a neighborhood of* c if there exists a neighborhood U of c and a number M such that $|f(x)|\leq M$ for all $x\in U\cap D$

(a) Suppose that f is bounded on a neighborhood of each x in D and that D is compact. Prove that f is bounded on D.

(b) Suppose that f is bounded on a neighborhood of each x in D and that D is not compact. Show that f is not necessarily bounded on D, even when f is continuous.

(c) Suppose that $f:[a,b]\to\mathbb{R}$ has a limit at each x in $[a,b]$. Prove that f is bounded on $[a,b]$.

(3) Prove that the function $f(x)=\frac{1}{x}$ on $[2,\infty)$ is uniformly continuous by verifying the $\epsilon-\delta$ property.

(4) Prove that $f(x)=\sqrt{x}$ is uniformly continuous on $[0,\infty)$.

(5) Let f and g be two real-valued functions that are uniformly continuous on a set D. Prove that $f+g$ is uniformly continuous on D.

(6) Find two real-valued functions f and g that are uniformly continuous on a set D, but such that their product $f\,g$ is not uniformly continuous on D.