

Quiz/HW 3

Please, write clearly and justify your steps to get credit for your work.

(2) [12 Pts] Mark each statement as True or False. If False, show a counter-example. If True, justify your answer.

(a) Every intersection of bounded and closed sets is compact.

True. Every intersection of bounded sets is bounded and every intersection of closed sets is closed. It follows that every intersection of bounded and closed sets is also a bounded and closed set. By the Heine-Borel theorem, this set is compact.

(b) The set $S = \{\frac{1}{n} : n \in \mathbb{N}\}$ is compact.

False. The set S is not closed since 0 is an accumulation point of S but $0 \notin S$.

(c) If S is unbounded then S has an accumulation point.

False. The set \mathbb{N} is unbounded yet has no accumulation points.

(d) If $S \subset \mathbb{R}$ is compact and x is an accumulation point of S , then $x \in S$.

True. If a set is compact then it is closed and it must contain all its accumulation points.

(e) If $S \subset \mathbb{R}$ is a compact, then there is at least one point in \mathbb{R} that is an accumulation point of S .

False. The set $S = \{1, 2, 3\}$ is compact since closed and bounded but it contains no accumulation points.

(f) If a set S has a maximum and a minimum, then S is a closed set.

False. The set $[0, 1) \cup (2, 3]$ has minimum ($x = 0$) and maximum ($x = 3$) but is not a closed set