

Quiz/HW 4

Please, write clearly and justify all your statements using the material covered in class to get credit for your work.

(1) [6 Pts] Let  $(s_n)$  be a sequence such that  $\lim_{n \rightarrow \infty} s_n = 0$  and  $(t_n)$  be a bounded sequence. Prove that the sequence  $(s_n t_n)$  is convergent.

*I will prove that  $\lim_{n \rightarrow \infty} s_n t_n = 0$ .*

*Since  $(t_n)$  is bounded, there is an  $M > 0$  such that  $t_n < M$  for all  $n \in \mathbb{N}$ .*

*Since  $\lim_{n \rightarrow \infty} s_n = 0$ , given any  $\epsilon > 0$ , there exists and  $N = N(\epsilon)$  such that  $|s_n| < \frac{\epsilon}{M}$  if  $n > N$ .*

*It follows that, given  $\epsilon > 0$ , there exists  $N = N(\epsilon)$  such that  $|s_n t_n| < \frac{\epsilon}{M} M = \epsilon$  if  $n > N$ .*

*This shows that  $\lim_{n \rightarrow \infty} s_n t_n = 0$ .*

(2)[2 Pts] Prove or give a counterexamples:

(a) If  $(s_n)$  and  $(t_n)$  are divergent sequences, then  $(s_n + t_n)$  diverges.

*FALSE. Let  $(s_n) = (n)$  and  $(t_n) = (-n)$ .  $(s_n + t_n) = 0$  convergent.*

(b) If  $(s_n)$  is convergent and  $(t_n)$  is bounded, then  $(s_n t_n)$  converges.

*FALSE. Let  $(s_n) = 1$  and  $(t_n) = (-1)^n$ .  $(s_n t_n) = 1$  divergent.*