Math 3333   Name:

Quiz/HW 4

Please, write clearly and justify all your statements using the material covered in class to get credit for your work.

(1) [4 Pts] Mark each statement as True or False. If False, show a counter-example. If True, justify your answer.

   (a) The set $S = \{\frac{1}{n} : n \in \mathbb{N}\}$ is compact.

   (b) If $S \subset \mathbb{R}$ is compact and x is an accumulation point of S, then $x \in S$.

   (c) If $S \subset \mathbb{R}$ is a compact, then there is at least one point in $\mathbb{R}$ that is an accumulation point of S.

   (d) If a set S has a maximum and a minimum, then S is a compact set.

(2) [4 Pts] Let (s_n) be a sequence such that $\lim_{n \to \infty} s_n = 0$ and (t_n) be a bounded sequence. Prove that the sequence $(s_n t_n)$ is convergent.

(3)[2 Pts] Prove or give a counterexamples:
   (a) If (s_n) and (t_n) are divergent sequences, then $(s_n + t_n)$ diverges.

   (b) If (s_n) is convergent and (t_n) is bounded, then $(s_n t_n)$ converges.