Math 3333 – Spring 2025 Name: SOLUTION

Quiz 5

Please, write clearly and justify all your statements using the material covered in class to get credit for your work.

(1)[5Pts] Prove that the sequence below is monotone and bounded. Next find its limit.

$$s_1 = 2, \quad s_{n+1} = \frac{1}{4}(2s_n + 7), \quad n \geq 1.$$

Proof. Note that $s_1 = 2$, $s_2 = \frac{1}{4}(4+7) = \frac{11}{4} > 2$.
Claim: $s_n \leq 4$. *Proof by induction:*
$s_1 = 2 \leq 4$.
Assume $s_n \leq 4$.
Then $s_{n+1} = \frac{1}{4}(s_n + 7) \leq \frac{11}{4} < 4$.

Claim: $s_{n+1} \geq s_n$. *Proof by induction:*
$s_2 > s_1$.
Assume $s_{n+1} \geq s_n$.
Then $s_{n+2} = \frac{1}{4}(2s_{n+1} + 7) \geq \frac{1}{4}(2s_n + 7) = s_{n+1}$.

Since (s_n) *is monotone nondecreasing and bounded above, then it is convergent. Thus*

$$s = \lim s_{n+1} = \lim \frac{1}{4}(2s_n + 7) = \frac{1}{4}(2s + 7).$$

Hence $4s = 2s + 7$ *and* $s = \frac{7}{2}$.

(1) [3Pts] Prove or give a counterexample:

(a) Every monotone sequence converges.

FALSE. $(s_n) = (n)$ *is monotone but not convergent.*

(b) If (a_n) and (b_n) are monotone sequences, then $(c_n) = (a_n + b_n)$ is also a monotone sequence.

FALSE. Let $(a_n) = (1, 2, 2, 2, \dots)$ *and* $(b_n) = (2, 2, 1, 1, \dots)$. *Then* $(c_n) = (a_n + b_n) = (3, 4, 3, 3, \dots)$ *is not monotone.*

(c) If (a_n) and (b_n) are monotone non-decreasing sequences, then $(c_n) = (a_n + b_n)$ is also a monotone non-decreasing sequence.

TRUE. If $a_{n+1} \geq a_n$ *and* $b_{n+1} \geq b_n$, *then* $c_{n+1} = a_{n+1} + b_{n+1} \geq a_n + b_n = c_n$.