

Quiz 6

Please, write clearly and justify all your statements using the material covered in class to get credit for your work.

(1)[4pts] Find an example of a sequence (s_n) of real numbers satisfying each set of properties.

(a) Cauchy, but not monotone.

$$s_n = (-1)^n \frac{1}{n}$$

(b) Monotone, but not Cauchy.

$$s_n = n$$

(c) Bounded, but not Cauchy.

$$s_n = (-1)^n$$

(d) (s_n) converges to 0 but $\sum s_n$ is not a convergent series.

$$s_n = \frac{1}{n}$$

(2)[4pts] Let (a_n) be a sequence of nonnegative real numbers. Prove that $\sum a_n$ converges iff the sequence of partial sums is bounded.

Proof. Since $a_n \geq 0$ for all n , then the sequence of partial sums $(s_n) = (\sum_{k \leq n} a_k)$ is monotone nondecreasing. It follows by the Monotone Convergence Theorem of sequences that (s_n) converges iff it is bounded.