

Quiz 7

(a)[2 Pts] Let

$$h(x) = \frac{x^2 + x - 2}{x + 2}$$

Is h continuous at $x = -2$? Justify your answer.

Since the denominator of $h(x)$ vanishes at $x = -2$, then h is not defined at $x = -2$ and, hence, it is not continuous at $x = -2$.

(b)[3 Pts] Let

$$f(x) = \begin{cases} \frac{x^2+x-2}{x+2} & \text{if } x \neq -2 \\ 0 & \text{if } x = -2. \end{cases}$$

Prove that f is discontinuous at $x = -2$.

We observe that

$$\lim_{x \rightarrow -2} f(x) = \lim_{x \rightarrow -2} \frac{x^2 + x - 2}{x + 2} = \lim_{x \rightarrow -2} \frac{(x - 1)(x + 2)}{x + 2} = -3.$$

Since $0 = f(-2) \neq \lim_{x \rightarrow -2} f(x)$, then f is discontinuous at $x = -2$.

(c)[3 Pts] Define a so that g below will be continuous at $x = -2$. Prove the continuity at $x = -2$.

$$g(x) = \begin{cases} \frac{x^2+x-2}{x+2} & \text{if } x \neq -2 \\ a & \text{if } x = -2. \end{cases}$$

It follows from part (b) that if we set $a = g(-2) = -3$, then

$$\lim_{x \rightarrow -2} g(x) = -3 = g(-2).$$

Hence we need to assign $a = g(-2) = -3$ to ensure that g is continuous at $x = -2$.