Math 3333 Name: SOLUTION

Quiz 8

1) Determine the following limit

$$\lim_{x \to 1^-} \frac{x-1}{|x-1|}$$

(a) using the sequential definition;
(b) using the $\epsilon - \delta$ definition.

(a) Let x_n be a sequence converging to 1^-. That is, there is exists an $N > 0$ such that $x_n < 1$ if $n > N$. Then

$$\lim_{x \to 1^-} \frac{x-1}{|x-1|} = \lim_{n > N} \frac{x_n - 1}{|x_n - 1|} = \lim_{n > N} \frac{x_n - 1}{1 - x_n} = -1.$$

(b) Given $\epsilon > 0$, let δ be any positive quantity. Then, if $1 - \delta < x < 1$, we have that

$$\left| \frac{x-1}{|x-1|} + 1 \right| = \left| \frac{x-1}{(1-x)} + 1 \right| = 0 < \epsilon.$$

(2) Let $f : D \to \mathbb{R}$ be continuous at $c \in D$. Prove that there exists an $M > 0$ and a neighborhood U of c such that $|f(x)| \leq M$ for all $x \in U \cap D$.

Proof. *Since f is continuous at c, there exists a $\delta > 0$ such that if $|x - c| < \delta$ and $x \in D$, then $|f(x) - f(c)| < 1$. This implies that, for all $x \in D$ such that $|x - c| < \delta$, we have that $|f(x)| \leq 1 + |f(c)| := M$.*