Math 3333 Name:

QUIZ 9

(I) Prove that the function $f(x) = \frac{1}{x}$ on $(2, \infty)$ is uniformly continuous by verifying the $\epsilon - \delta$ property. **NOTE: you need to explicitly derive an expression of δ as a function of ϵ.**

Proof. *Observe that, for any $x, y \in [2, \infty)$, $\frac{1}{x}, \frac{1}{y} \leq \frac{1}{2}$. Hence*

$$|\frac{1}{x} - \frac{1}{y}| = \frac{|y-x|}{xy} \leq \frac{|y-x|}{4}.$$

It follows that, given any $\epsilon > 0$, if we set $\delta = 4\epsilon$, then $|x-y| < \delta$ implies that $|\frac{1}{x} - \frac{1}{y}| < \epsilon$. This proves that the function $f(x) = \frac{1}{x}$ on $(2, \infty)$ is uniformly continuous.

(II) Prove that the function $f(x) = \frac{1}{\sqrt{x}}$ on $(2, \infty)$ is uniformly continuous by verifying the $\epsilon - \delta$ property. **NOTE: you need to explicitly derive an expression of δ as a function of ϵ.**

Proof. *Observe that, for any $x, y \in [2, \infty)$, $\frac{1}{\sqrt{x}}, \frac{1}{\sqrt{y}} \leq \frac{1}{2}$. Hence*

$$|\frac{1}{\sqrt{x}} - \frac{1}{\sqrt{y}}| = \frac{|\sqrt{y} - \sqrt{x}|}{\sqrt{xy}} = \frac{|y-x|}{|\sqrt{y} + \sqrt{x}|\sqrt{xy}} \leq \frac{|y-x|}{4\sqrt{2}}.$$

It follows that, given any $\epsilon > 0$, if we set $\delta = 4\sqrt{2}\epsilon$, then $|x-y| < \delta$ implies that $|\frac{1}{\sqrt{x}} - \frac{1}{\sqrt{y}}| < \epsilon$.