Math 3339 Name: SOLUTION

HW # 10

(1)[5 Pts] Let the test statistic W have a t distribution when H_0 is true. Give the significance level for each of the following situations

(i) $H_1 : \mu > m_0$, df=15, rejection region $t \geq 2.947$

(ii) $H_1 : \mu < m_0$, df=24, rejection region $t \leq -2.500$

(iii) $H_1 : \mu \neq m_0$, df=30, rejection region $t \leq -1.697$ or $t \geq 1.697$

Using R (i) `> 1-pt(2.947,15) = 0.004997084`
Thus $\alpha = 0.005$
(ii) `> pt(-2.500,24) = 0.009827088`
Thus $\alpha = 0.01$,
(iii) `pt(-1.697,30) = 0.05002492`
Thus $\alpha/2 = 0.05, \alpha = 0.1$

(2)[5 Pts] A tire company claims that the average mileage of a certain brand of tires can last is 29,200 miles. A sample of $n = 22$ tires is taken at random to assess their mileage, resulting in the sample mean $\overline{x} = 29,132$ and sample variance $s^2 = 2,236$. Assuming that the distribution is normal:

(a) test the hypothesis that the true average mileage of the tires is different from 29,200 miles. Choose confidence level $\alpha = 0.01$.

(b) find a 99 percent confidence interval for μ.

(a) We test the hypothesis

$$H_0 : \mu = 29,200;$$

$$H_1 : \mu \neq 29,200.$$

Test statistic (t-test) $W = \frac{\bar{X}-\mu_0}{s/\sqrt{n}} = \frac{29,132-29,200}{\sqrt{2,236}/\sqrt{22}} = -6.745$ For $\alpha = 0.01$, $\alpha/2 = 0.005$, $t_{0.005,21} = \texttt{qt}(0.995, 21) = 2.883$

Since, $W = -6.745 < -2.832$, we reject H_0 at significance level $\alpha = 0.01$.

Note: the p=value is `2*pt(-6.745,21) = 1.131726e-06`

(b) $CI: \ 29132 \pm t_{0.005;21}\frac{\sqrt{2236}}{\sqrt{22}} = 29132 \pm 2.831\frac{\sqrt{2236}}{\sqrt{22}} = [29103.4, 29160.6]$

(3)[5 Pts] Lightbulbs of a certain type are advertised as having an average lifetime of 750 hours. A random sample of 50 bulbs was selected, the lifetime of each bulb determined finding that the sample average lifetime is 738.5 with sample standard deviation 38.2. Test the hypothesis that the true average lifetime is smaller than what is advertised using significance level $\alpha = 0.05$ and $\alpha = 0.01$.

We test the hypothesis

$H_0 : \mu = 750$;

$H_1 : \mu < 750$.

Test statistic (z-test) $W = \frac{\bar{X}-\mu_0}{s/\sqrt{n}} = \frac{738.5-750}{38.2/\sqrt{50}} = -2.130$

For $\alpha = 0.05$, $z_{0.05} = 1.645$; *for* $\alpha = 0.01$, $z_{0.01} = 2.326$

Since, $W = -2.130 < -1.645$, *we reject* H_0 *at significance level* $\alpha = 0.05$.

Since, $W = -2.130 > -2.326$, *we fail to reject* H_0 *at significance level* $\alpha = 0.01$.

p-value: $P(z < -2.130) =$ *(using R:* `pnorm(-2.130)`*) = 0.017*

(4)[5 Pts] A rubber compound were tested for tensile strength and the following values were found

$$32, 30, 31, 33, 32, 30, 29, 34, 32, 31$$

Assuming that the population is normally distributed, test the hypothesis that the average tensile strength is different from 31. Use $\alpha = 0.05$. Calculate the p-value of the test.

We test the hypothesis

$H_0 : \mu = 31$;

$H_1 : \mu \neq 31$.

Test statistic (t-test) $W = \frac{\bar{X}-\mu_0}{s/\sqrt{n}} = \frac{31.4-31}{1.506/\sqrt{10}} = 0.84017$

For $\alpha = 0.05$, $\alpha/2 = 0.025$, $t_{0.025,9} = 2.26216$

Since, $W = 0.84017 < 2.26216$, *we fail to reject* H_0 *at significance level* $\alpha = 0.05$.

p-value $= 2P(t < -0.84017) =$ (*using R:* `2*pt(-0.84017,9)`) $= 0.4225706$

R solution

```
> x <-c(32, 30, 31, 33, 32, 30, 29, 34, 32, 31)
> t.test(x,mu=31,alternative = "two.sided", conf.level = 0.95)


One Sample t-test

data:  x
t = 0.84017, df = 9, p-value = 0.4226
alternative hypothesis:  true mean is not equal to 31
95 percent confidence interval:
30.323 32.477
sample estimates:
mean of x
31.4
```

(5)[5 Pts] Using the same data as in Problem 4 and still assuming that the population is normally distributed, test the hypothesis that the average tensile strength is larger than 31. Use $\alpha = 0.05$. Calculate the p-value of the test.

We test the hypothesis

$H_0 : \mu = 31;$

$H_1 : \mu > 31.$

```
> x <-c(32, 30, 31, 33, 32, 30, 29, 34, 32, 31)
> t.test(x,mu=31,alternative = "greater", conf.level = 0.95)
```

```
One Sample t-test

data:  x
t = 0.84017, df = 9, p-value = 0.2113
alternative hypothesis:  true mean is not equal to 31
95 percent confidence interval:
30.323 32.477
sample estimates:
mean of x
31.4
```

Also in this case, p-value is larger than 0.05, hence we fail to reject H_0 at significance level $\alpha = 0.05$.

(6)[5 Pts] State DMV records indicate that of all vehicles undergoing emissions testing during the previous year, 70% passed on the first try. A random sample of 200 cars tested in a particular county during the current year yields 160 that passed on the initial test. Does this suggest that the true proportion for this county during the current year differs from the previous statewide proportion? Test the relevant hypotheses using $\alpha = 0.05$.

SOLUTION

Hypothesis test about proportion. TWO TAIL TEST

We test $H_0 : p = 0.70$ against $H_1 : p \neq 0.70$ with $\alpha = 0.05$.
Data: $\hat{p} = \frac{160}{200} = 0.80$
Discussion: this is a test about proportion We need to use a z-test.
Test statistic $z = \frac{\hat{p}-p_0}{\sqrt{p_0(1-p_0)/n}} = \frac{0.80-0.70}{\sqrt{(0.7)(0.3)/200}} = 3.086$
Using R, we have $z_{\alpha/2} = \texttt{qnorm}(1 - \alpha/2)$
Hence:
$z_{\alpha/2} = z_{0.025} = \texttt{qnorm}(0.975) = 1.959964$
Since $z > z_{0.025} = 1.960$, then H_0 can be rejected.

p-value: $2\,P(Z < -|z|)$. Hence:

```
> 2*pnorm(-3.086) = 0.002028688
```

***

We can alternatively run the R command prop.test

```
prop.test(160,200,p=0.70,alternative = "two.sided",correct = FALSE)
```

you find

```
X-squared = 9.5238, df = 1, p-value = 0.002028
```

Since the p-value is less than $\alpha = 0.05$, we reject H_0