HW #10

(1) Two rubber compounds were tested for tensile strength and the following values were found

$$A: \; 32, 30, 33, 32, 29, 34, 32$$
$$B: \; 33, 35, 36, 37, 35, 34$$

Under the assumption that the two populations are normally distributed, test the hypothesis that the average tensile strength of the two rubber compounds is different using significance level $\alpha = 0.01$ and $\alpha = 0.05$.

We test $H_0 : \mu_A = \mu_B$ against $H_1 : \mu_A \neq \mu_B$ with $\alpha = 0.05$ and $\alpha = 0.01$.

Since the populations are normal and the variance is unknown, we run a 2-population t-test

We first find the sample means $\bar{x}_A = 31.7$ and $\bar{x}_B = 35.0$ and the sample variances $s_A^2 = 2.9$ and $s_B^2 = 2.0$.

We compute the pooled variance $s_p^2 = \frac{(n_A - 1)s_A^2 + (n_B - 1)s_B^2}{n_A + n_B - 2} = 2.49$

Test statistic (Student t pdf):

$$t = \frac{\bar{x}_A - \bar{x}_B}{\sqrt{\frac{s_p^2}{n_A} + \frac{s_p^2}{n_B}}} = \frac{31.7 - 35.0}{\sqrt{\frac{2.49}{7} + \frac{2.49}{6}}} = -3.759$$

Rejection regions: $|t| > t_{0.025;11} = 2.201$, $|t| > t_{0.005;11} = 3.106$

Since $t < -t_{0.025;11} = -2.201$ and $t < -t_{0.005;11} = -3.106$ then $\underline{H_0 \text{ is REJECTED}}$ at significance levels $\alpha = 0.05$ and $\alpha = 0.01$.

One can solve using R with

```
> t.test(x,y,alternative = "two.sided", paired = FALSE,var.equal
= TRUE, conf.level = 0.95)
```

You find: p-value = 0.003266

Since the p-value is less than $\alpha = 0.01$ and $\alpha = 0.05$, then we reject H_0 at both significance levels.

(2) In comparing the times until failure (in hours) of two different types of light bulbs, we obtain the sample characteristics $n_1 = 45$, $\bar{x} = 984$, $s_x^2 = 8,742$ and $n_2 = 52$, $\bar{y} = 1,121$, $s_x^2 = 9,411$. Test the hypothesis that the average duration of the second type of light bulbs is higher than the first type. at significance level $\alpha = 0.05$.

We test $H_0 : \mu_1 \geq \mu_2$ against $H_1 : \mu_1 < \mu_2$ with $\alpha = 0.05$.
Since the population distribution is unknown and the population sizes are larger than 30, we will apply a 2-population z-test.
Test statistic (normal pdf):

$$W = \frac{\bar{x} - \bar{y}}{\sqrt{\frac{s_x^2}{n_1} + \frac{s_y^2}{n_2}}} = \frac{984 - 1121}{\sqrt{\frac{8742}{45} + \frac{9411}{52}}} = -7.07$$

Since $W = -7.07 < -z_{0.05} = -1.645$, then $\underline{H_0 \text{ is REJECTED}}$.

(3) A sample of 12 radon detectors of a certain type was selected, and each was exposed to 100 pCi/L of radon. The resulting readings were as follows:

$$105.6, 90.9, 91.2, 96.9, 96.5, 91.3, 100.1, 105.0, 99.6, 107.7, 103.3, 92.4$$

(a) Apply the Shapiro-Wilk test to verify that data can be modeled using the Normal distribution.
(b) Does this data suggest that the population mean reading under these conditions differs from 100? State and test the appropriate hypotheses using significance level $\alpha = 0.05$.
(c) Suppose that prior to the experiment a value of $\sigma = 7.5$ had been assumed. How many determinations would then have been appropriate to obtain $\beta = 0.10$ for the alternative hypothesis $\mu_1 = 95$?

(a) We run the Shapiro-Wilk normality test

```
> x <-c(105.6, 90.9, 91.2, 96.9, 96.5, 91.3, 100.1, 105.0, 99.6,
107.7, 103.3, 92.4)
> shapiro.test(x)

	Shapiro-Wilk normality test
```

```
   data:  x
W = 0.9115, p-value = 0.223
```

Since p-value ¿ 0.05, the data can be assumed to be normal.
(b)
We test $H_0 : \mu = 100$ against $H_1 : \mu \neq 100$ with $\alpha = 0.05$.
Since the data is normal and the variance is unknown, we apply a t=test.
We find the sample mean $\bar{x} = 98.38$ and the sample variance $s^2 = 37.33$.
Test statistic (Student t pdf):

$$W = \frac{\bar{x} - 100}{\sqrt{\frac{s^2}{n}}} = \frac{98.38 - 100}{\sqrt{\frac{37.33}{12}}} = -0.92$$

Rejection regions: $|t| > t_{0.025,11} = 2.20$,
Since $z > -t_{0.025,11} = -2.20$ then $\underline{H_0 \text{ is ACCEPTED}}$ at significance levels $\alpha = 0.05$.
(c) We apply the following formula to determine the sample size

$$n = \frac{\sigma^2(z_{\alpha/2} + z_\beta)^2}{(\mu_0 - \mu_1)^2} = \frac{(7.5)^2(1.96 + 1.28)^2}{5^2} = 23.6$$

Hence we choose $n = 24$.

(4) Subjects in a study included a sample of 37 male soccer players whose mean body mass index (BMI) was 25.21 with a sample standard deviation of 1.67 and a sample of 24 male rugby players whose mean BMI was 27.15 with a sample standard deviation of 2.64. Is there sufficient evidence for one to claim that, in general, rugby players have a different BMI than soccer players? Let $\alpha = 0.01$. You can assume that data are normal and that the true variances of the populations are the about same.

We test $H_0 : \mu_r = \mu_s$ against $H_1 : \mu_r \neq \mu_s$ with $\alpha = 0.01$.
Since the populations are normal and the variance is unknown, we run a 2-population t-test
Data: $n_r = 24$, $\bar{x}_r = 27.15$, $s_r = 2.64$; $n_s = 37$, $\bar{x}_s = 25.21$, $s_s = 1.67$. The sample variance is $s_p^2 = \frac{(n_r-1)s_r^2 + (n_s-1)s_s^2}{n_r + n_s - 2} = 4.419$.

Test statistic (Student t pdf):

$$t = \frac{\bar{x}_r - \bar{x}_s}{\sqrt{\frac{s_p^2}{n_r} + \frac{s_p^2}{n_s}}} = \frac{27.15 - 25.21}{\sqrt{\frac{4.419}{24} + \frac{4.419}{37}}} = 3.521$$

Rejection region: $t > t_{0.005;60} = 2.660$
Since $t > t_{0.005;60} = 2.660$, then $\underline{H_0 \text{ is REJECTED}}$.

(5) A study found that among 2430 boys ages 7 to 12 years, 450 were overweight or obese. On the basis of this study, can we conclude that more than 15 percent of the boys ages 7 to 12 in the sampled population are obese or overweight? Let $\alpha = 0.01$.

We test $H_0 : p \leq 0.15$ against $H_1 : p > 0.15$ with $\alpha = 0.01$.
We will run a proportion test.
Data: $n = 2430$, $x = 450$, $\hat{p} = \frac{450}{2430} = 0.185$.
Test statistic (Standard Normal pdf):

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{0.185 - 0.150}{\sqrt{\frac{(0.15)(0.85)}{2430}}} = 4.857$$

Rejection region: $z > z_{0.01} = 2.326$
Since $z > z_{0.01}$, then $\underline{H_0 \text{ is REJECTED}}$.

We can also solve the problem using R:
```
prop.test(x=450,n=2430,p=0.15,alternative = "greater",correct
= FALSE))
```
This will give p-value = 5.946e-07 so that H_0 is REJECTED for $\alpha = 0.01$.

(6) A study is conducted to evaluate the analgesic effectiveness of a daily dose of oral methadone in patients with chronic neuropathic pain. The researchers used a scale [0,100] with higher number indicating higher pain. Each subject took either 20 mg of methadone or placebo each day for 5 days, without knowing which treatment they were taking. The following table gives the mean maximum pain intensity scores for the 5 days for each subject. Do these data provide sufficient evidence to indicate tha the maximum pain intensity is lower on days when methadone is taken? Let $\alpha = 0.05$.

Subject	Methadone	Placebo
1	29.8	57.2
2	73.0	69.8
3	98.6	98.2
4	58.8	62.4
5	60.6	67.2
6	57.2	70.6
7	57.2	67.8
8	89.2	95.6
9	97.0	98.4
10	49.8	63.2
11	37.0	63.6

We note that data are dependent since the same subject is used to derive two measurements. Therefore we need to run a paired sample t-test

We compare max pain intensity with metadone (m) or placebo (p). We consider the paired differences $d_i = m_i - p_i$

We test $H_0 : \mu_d \geq 0$ against $H_1 : \mu_d < 0$ with $\alpha = 0.05$.

Data: $n = 11$, $\bar{d} = \frac{1}{11} \sum_{i=1}^{11} d_i = -9.618$, $s_d^2 = 102.204$.

Test statistic (Student t pdf):

$$t = \frac{\bar{d} - \mu_d}{s_{\bar{d}}} = \frac{\bar{d} - \mu_d}{s_d/\sqrt{n}} = \frac{-9.618}{\sqrt{\frac{102.204}{11}}} = -3.155$$

Rejection region: $t < -t_{0.05;10} = -1.812$

Since $t < -t_{0.05;10} = -1.812$, then $\underline{H_0 \text{ is REJECTED}}$.

We can solve in R

```
> x <- c(29.8,73.0,98.6,58.8,60.6,57.2,57.2,89.2,97.0,49.8,37.0)
> y<-c(57.2,69.8,98.2,62.4,67.2,70.6,67.8,95.6,98.4,63.2,63.6)
> t.test(x,y,alternative = "less", paired = TRUE, var.equal = TRUE)
```

You will find p-value = 0.005119, hence H_0 is REJECTED at significance level 0.05.