Math 3339 – Fall 2020 Name:

HW #1

Please, write clearly and justify all your steps, to get proper credit for your work.

(1) In a class of 125 students, the instructor gave 40 A's, 30 B's, 35 C's, 15 D's and 5 F's. Compute the relative frequency of the 5 events. Compute the relative frequency of getting a grade that is better than C.

Solution.
$P(A) = 40/125 = 8/25$, $P(B) = 30/125 = 6/25$, $P(C) = 35/125 = 7/25$, $P(D) = 15/125 = 3/25$, $P(F) = 5/125 = 1/25$.
$P(\text{better than C}) = P(B) + P(A) = 14/25$

(2) Let $S = A \cup B$, $P(A) = 0.6$, $P(B) = 0.8$.

(a) Find $P(A \cap B)$

(b) Find $P(A^c \cup B^c)$

Solution.
(a) $1 = P(S) = P(A \cup B) = P(A) + P(B) - P(A \cap B)$
Hence $P(A \cap B) = P(A) + P(B) - P(A \cup B) = 1.4 - 1 = 0.4$
(b) $P(A^c \cup B^c) = P((A \cap B)^c) = 1 - P(A \cap B) = 0.6$

(3) Let $P(A \cap B) = 0.2$, $P(A) = 0.5$, $P(B) = 0.4$.

(a) Find $P(A \cup B)$

(b) Find $P(A^c \cup B^c)$

(c) Find $P(A^c \cap B)$

Solution.
(a) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.5 + 0.4 - 0.2 = 0.7$
(b) $P(A^c \cup B^c) = P((A \cap B)^c) = 1 - P(A \cap B) = 1 - 0.2 = 0.8$
(c) $P(A^c \cap B) = P(B \setminus A) = P(B) - P(A \cap B) = 0.4 - 0.2 = 0.2$

(4) How many different ways can you rearrange the letters of the following words

(a) OLSEN

(b) CALCUTTA

Solution.

(a) The 5 letters are distinct. Number of permutations is:

$$(5)(4)(3)(2)(1) = 5!$$

(b) Of the 8 letters, the letters A, C and T are repeated. Number of permutations is:

$$\frac{8!}{2!\,2!\,2!}$$

(5) A bowl contains 20 chips, of which 9 are red, 8 are blue and 3 are white. Six chips are drawn at random and without replacement.

(i) Compute the probability that each of the 6 chips is red.

Solution.

$$P(6 \text{ red}) = \frac{\binom{9}{6}}{\binom{20}{6}}$$

(ii) Compute the probability that 3 chips are red and 3 chips are blue.

Solution.

$$P(3 \text{ red, } 3 \text{ blue}) = \frac{\binom{9}{3}\binom{8}{3}}{\binom{20}{6}}$$