Math 3339 Name:

HW #2

Please, write clearly and justify all your steps, to get proper credit for your work.

(1) Two cards are drawn successively and without replacement from a 52-card deck of playing cards. Compute the probability of drawing:

(i) two hearts;

(ii) a heart on the first draw, a club on the second draw;

(iii) a heart on the first draw, an ace on the second draw.

(2) From a regular deck of 52 playing cards, cards are drawn successively and without replacement. Compute the probability that the third spade appears on the sixth draw.

(3) A survey organization asked respondents from 3 different geographical regions what they views were on a certain topic. The answer are reported below.

	East	Midwest	West
Pessimistic	100	90	110
Optimistic	40	70	90
Total	140	160	200

(i) What is the probability that a randomly selected respondent is pessimistic?

(ii) What is the conditional probability that a respondent from the Midwest is optimistic?

(iii) What is the conditional probability that a respondent who is optimistic comes from the Midwest?

(iv) Are the views of the respondents independent on the geographical regions? Justify your answer. If no, specify what the numbers for the West region would have been, had the two factors been independent.

(4) A disease in a bat population has a prevalence of 0.1%. To test for the presence of the disease, a screening test was designed with a reported sensitivity of 92% and a reported specificity of 85%. We a randomly selected bat from this population and perform the screening test. What is the probability that the bat is affected by the disease if the test returns positive? What is the probability that the bat is not affected by the disease if the test returns negative?

(5) The surfaces of human red blood cells ("erythrocytes") are coated with antigens that are classified into four disjoint blood types: O, A, B, and AB. Each type is associated with blood serum antibodies for the other types, that is

1. Type O blood contains both A and B antibodies. (This makes Type O the "universal donor", but capable of receiving only Type O.)
2. Type A blood contains only B antibodies.
3. Type B blood contains only A antibodies.
4. Type AB blood contains neither A nor B antibodies. (This makes Type AB the "universal recipient", but capable of donating only to Type AB.)

In addition, blood is also classified according to the presence (+) or absence (-) of Rh factor. Hence there are eight distinct blood groups corresponding to this joint classification system: $O^+, O^-, A^+, A^-, B^+, B^-, AB^+, AB^-$. According to the American Red Cross, the U.S. population has the following blood group relative frequencies

Blood type	Rh+	Rh-	*Totals*
O	0.384	0.077	0.461
A	0.323	0.065	0.388
B	0.094	0.017	0.111
AB	0.032	0.007	0.039
Totals	0.833	0.166	0.999

From these table, we can calculate, for instance, the following probabilities:
$P(\text{Blood type } O^+) = 0.384$,
$P(\text{Blood type } O) = P(\text{Blood type } O^+) + P(\text{Blood type } O^-) = 0.461$,
$P(A \text{ antibodies}) = P(\text{Blood type } O) + P(\text{Blood type } B) = 0.461 + 0.111 = 0.572$.
$P(B \text{ antibodies}) = P(\text{Blood type } O) + P(\text{Blood type } A) = 0.461 + 0.388 = 0.849$.
Answer the following questions:
(a) Is having A antibodies independent of having B antibodies?
(b) Is having B antibodies independent of Rh^+?