23. The computers of six faculty members in a certain department are to be replaced. Two of the faculty members have selected laptop machines and the other four have chosen desktop machines. Suppose that only two of the setups can be done on a particular day, and the two computers to be set up are randomly selected from the six (implying 15 equally likely outcomes; if the computers are numbered 1, 2, . . . , 6, then one outcome consists of computers 1 and 2, another consists of computers 1 and 3, and so on).

a. What is the probability that both selected setups are for laptop computers?

b. What is the probability that both selected setups are desktop machines?

c. What is the probability that at least one selected setup is for a desktop computer?

d. What is the probability that at least one computer of each type is chosen for setup?

24. Show that if one event A is contained in another event B (i.e., A is a subset of B), then $P(A) \leq P(B)$. [*Hint:* For such A and B, A and $B \cap A'$ are disjoint and $B = A \cup (B \cap A')$, as can be seen from a Venn diagram.] For general A and B, what does this imply about the relationship among $P(A \cap B)$, $P(A)$ and $P(A \cup B)$?

25. The three most popular options on a certain type of new car are a built-in GPS (A), a sunroof (B), and an automatic transmission (C). If 40% of all purchasers request A, 55% request B, 70% request C, 63% request A or B, 77% request A or C, 80% request B or C, and 85% request A or B or C, determine the probabilities of the following events. [*Hint:* "A or B" is the event that at least one of the two options is requested; try drawing a Venn diagram and labeling all regions.]

a. The next purchaser will request at least one of the three options.

b. The next purchaser will select none of the three options.

c. The next purchaser will request only an automatic transmission and not either of the other two options.

d. The next purchaser will select exactly one of these three options.

26. A certain system can experience three different types of defects. Let A_i ($i = 1,2,3$) denote the event that the system has a defect of type i. Suppose that

$$P(A_1) = .12 \quad P(A_2) = .07 \quad P(A_3) = .05$$
$$P(A_1 \cup A_2) = .13 \quad P(A_1 \cup A_3) = .14$$
$$P(A_2 \cup A_3) = .10 \quad P(A_1 \cap A_2 \cap A_3) = .01$$

a. What is the probability that the system does not have a type 1 defect?

b. What is the probability that the system has both type 1 and type 2 defects?

c. What is the probability that the system has both type 1 and type 2 defects but not a type 3 defect?

d. What is the probability that the system has at most two of these defects?

27. An academic department with five faculty members—Anderson, Box, Cox, Cramer, and Fisher—must select two of its members to serve on a personnel review committee. Because the work will be time-consuming, no one is anxious to serve, so it is decided that the representative will be selected by putting the names on identical pieces of paper and then randomly selecting two.

a. What is the probability that both Anderson and Box will be selected? [*Hint:* List the equally likely outcomes.]

b. What is the probability that at least one of the two members whose name begins with C is selected?

c. If the five faculty members have taught for 3, 6, 7, 10, and 14 years, respectively, at the university, what is the probability that the two chosen representatives have a total of at least 15 years' teaching experience there?

28. In Exercise 5, suppose that any incoming individual is equally likely to be assigned to any of the three stations irrespective of where other individuals have been assigned. What is the probability that

a. All three family members are assigned to the same station?

b. At most two family members are assigned to the same station?

c. Every family member is assigned to a different station?

2.3 Counting Techniques

When the various outcomes of an experiment are equally likely (the same probability is assigned to each simple event), the task of computing probabilities reduces to counting. Letting N denote the number of outcomes in a sample space and $N(A)$ represent the number of outcomes contained in an event A,

$$P(A) = \frac{N(A)}{N} \tag{2.1}$$

If a list of the outcomes is easily obtained and N is small, then N and $N(A)$ can be determined without the benefit of any general counting principles.

There are, however, many experiments for which the effort involved in constructing such a list is prohibitive because N is quite large. By exploiting some

Next to each branch corresponding to a positive test result, the multiplication rule yields the recorded probabilities. Therefore, $P(B) = .00099 + .01998 = .02097$, from which we have

$$P(A_1 | B) = \frac{P(A_1 \cap B)}{P(B)} = \frac{.00099}{.02097} = .047$$

This result seems counterintuitive; the diagnostic test appears so accurate that we expect someone with a positive test result to be highly likely to have the disease, whereas the computed conditional probability is only .047. However, the rarity of the disease implies that most positive test results arise from errors rather than from diseased individuals. The probability of having the disease has increased by a multiplicative factor of 47 (from prior .001 to posterior .047); but to get a further increase in the posterior probability, a diagnostic test with much smaller error rates is needed. ■

EXERCISES Section 2.4 (45–69)

45. The population of a particular country consists of three ethnic groups. Each individual belongs to one of the four major blood groups. The accompanying *joint probability table* gives the proportions of individuals in the various ethnic group–blood group combinations.

		Blood Group			
		O	**A**	**B**	**AB**
Ethnic Group	**1**	.082	.106	.008	.004
	2	.135	.141	.018	.006
	3	.215	.200	.065	.020

Suppose that an individual is randomly selected from the population, and define events by A = {type A selected}, B = {type B selected}, and C = {ethnic group 3 selected}.

a. Calculate $P(A)$, $P(C)$, and $P(A \cap C)$.

b. Calculate both $P(A | C)$ and $P(C | A)$, and explain in context what each of these probabilities represents.

c. If the selected individual does not have type B blood, what is the probability that he or she is from ethnic group 1?

46. Suppose an individual is randomly selected from the population of all adult males living in the United States. Let A be the event that the selected individual is over 6 ft in height, and let B be the event that the selected individual is a professional basketball player. Which do you think is larger, $P(A | B)$ or $P(B | A)$? Why?

47. Return to the credit card scenario of Exercise 12 (Section 2.2), where A = {Visa}, B = {MasterCard}, $P(A) = .5$, $P(B) = .4$, and $P(A \cap B) = .25$. Calculate and interpret each of the following probabilities (a Venn diagram might help).

a. $P(B | A)$ **b.** $P(B' | A)$

c. $P(A | B)$ **d.** $P(A' | B)$

e. Given that the selected individual has at least one card, what is the probability that he or she has a Visa card?

48. Reconsider the system defect situation described in Exercise 26 (Section 2.2).

a. Given that the system has a type 1 defect, what is the probability that it has a type 2 defect?

b. Given that the system has a type 1 defect, what is the probability that it has all three types of defects?

c. Given that the system has at least one type of defect, what is the probability that it has exactly one type of defect?

d. Given that the system has both of the first two types of defects, what is the probability that it does not have the third type of defect?

49. The accompanying table gives information on the type of coffee selected by someone purchasing a single cup at a particular airport kiosk.

	Small	**Medium**	**Large**
Regular	14%	20%	26%
Decaf	20%	10%	10%

Consider randomly selecting such a coffee purchaser.

a. What is the probability that the individual purchased a small cup? A cup of decaf coffee?

b. If we learn that the selected individual purchased a small cup, what now is the probability that he/she chose decaf coffee, and how would you interpret this probability?

c. If we learn that the selected individual purchased decaf, what now is the probability that a small size was selected, and how does this compare to the corresponding unconditional probability of (a)?

50. A department store sells sport shirts in three sizes (small, medium, and large), three patterns (plaid, print, and stripe), and two sleeve lengths (long and short). The accompanying tables give the proportions of shirts sold in the various category combinations.

are the probabilities associated with 0, 1, and 2 defective components being in the batch under each of the following conditions?

a. Neither tested component is defective.

b. One of the two tested components is defective. [*Hint:* Draw a tree diagram with three first-generation branches for the three different types of batches.]

62. A company that manufactures video cameras produces a basic model and a deluxe model. Over the past year, 40% of the cameras sold have been of the basic model. Of those buying the basic model, 30% purchase an extended warranty, whereas 50% of all deluxe purchasers do so. If you learn that a randomly selected purchaser has an extended warranty, how likely is it that he or she has a basic model?

63. For customers purchasing a refrigerator at a certain appliance store, let A be the event that the refrigerator was manufactured in the U.S., B be the event that the refrigerator had an icemaker, and C be the event that the customer purchased an extended warranty. Relevant probabilities are

$$P(A) = .75 \quad P(B|A) = .9 \quad P(B|A') = .8$$
$$P(C|A \cap B) = .8 \quad P(C|A \cap B') = .6$$
$$P(C|A' \cap B) = .7 \quad P(C|A' \cap B') = .3$$

a. Construct a tree diagram consisting of first-, second-, and third-generation branches, and place an event label and appropriate probability next to each branch.

b. Compute $P(A \cap B \cap C)$.

c. Compute $P(B \cap C)$.

d. Compute $P(C)$.

e. Compute $P(A|B \cap C)$, the probability of a U.S. purchase given that an icemaker and extended warranty are also purchased.

64. The Reviews editor for a certain scientific journal decides whether the review for any particular book should be short (1–2 pages), medium (3–4 pages), or long (5–6 pages). Data on recent reviews indicates that 60% of them are short, 30% are medium, and the other 10% are long. Reviews are submitted in either Word or LaTeX. For short reviews, 80% are in Word, whereas 50% of medium reviews are in Word and 30% of long reviews are in Word. Suppose a recent review is randomly selected.

a. What is the probability that the selected review was submitted in Word format?

b. If the selected review was submitted in Word format, what are the posterior probabilities of it being short, medium, or long?

65. A large operator of timeshare complexes requires anyone interested in making a purchase to first visit the site of interest. Historical data indicates that 20% of all potential purchasers select a day visit, 50% choose a one-night visit, and 30% opt for a two-night visit. In addition, 10% of day visitors ultimately make a purchase, 30% of one-night visitors buy a unit, and 20% of those visiting for two nights decide to buy. Suppose a visitor is randomly selected and is found to have made a purchase. How likely is it that this person made a day visit? A one-night visit? A two-night visit?

66. Consider the following information about travelers on vacation (based partly on a recent Travelocity poll): 40% check work email, 30% use a cell phone to stay connected to work, 25% bring a laptop with them, 23% both check work email and use a cell phone to stay connected, and 51% neither check work email nor use a cell phone to stay connected nor bring a laptop. In addition, 88 out of every 100 who bring a laptop also check work email, and 70 out of every 100 who use a cell phone to stay connected also bring a laptop.

a. What is the probability that a randomly selected traveler who checks work email also uses a cell phone to stay connected?

b. What is the probability that someone who brings a laptop on vacation also uses a cell phone to stay connected?

c. If the randomly selected traveler checked work email and brought a laptop, what is the probability that he/she uses a cell phone to stay connected?

67. There has been a great deal of controversy over the last several years regarding what types of surveillance are appropriate to prevent terrorism. Suppose a particular surveillance system has a 99% chance of correctly identifying a future terrorist and a 99.9% chance of correctly identifying someone who is not a future terrorist. If there are 1000 future terrorists in a population of 300 million, and one of these 300 million is randomly selected, scrutinized by the system, and identified as a future terrorist, what is the probability that he/she actually is a future terrorist? Does the value of this probability make you uneasy about using the surveillance system? Explain.

68. A friend who lives in Los Angeles makes frequent consulting trips to Washington, D.C.; 50% of the time she travels on airline #1, 30% of the time on airline #2, and the remaining 20% of the time on airline #3. For airline #1, flights are late into D.C. 30% of the time and late into L.A. 10% of the time. For airline #2, these percentages are 25% and 20%, whereas for airline #3 the percentages are 40% and 25%. If we learn that on a particular trip she arrived late at exactly one of the two destinations, what are the posterior probabilities of having flown on airlines #1, #2, and #3?Assume that the chance of a late arrival in L.A. is unaffected by what happens on the flight to D.C. [*Hint:* From the tip of each first-generation branch on a tree diagram, draw three second-generation branches labeled, respectively, 0 late, 1 late, and 2 late.]

69. In Exercise 59, consider the following additional information on credit card usage:

70% of all regular fill-up customers use a credit card.
50% of all regular non-fill-up customers use a credit card.
60% of all plus fill-up customers use a credit card.
50% of all plus non-fill-up customers use a credit card.

75. One of the assumptions underlying the theory of control charting (see Chapter 16) is that successive plotted points are independent of one another. Each plotted point can signal either that a manufacturing process is operating correctly or that there is some sort of malfunction. Even when a process is running correctly, there is a small probability that a particular point will signal a problem with the process. Suppose that this probability is .05. What is the probability that at least one of 10 successive points indicates a problem when in fact the process is operating correctly? Answer this question for 25 successive points.

76. In October, 1994, a flaw in a certain Pentium chip installed in computers was discovered that could result in a wrong answer when performing a division. The manufacturer initially claimed that the chance of any particular division being incorrect was only 1 in 9 billion, so that it would take thousands of years before a typical user encountered a mistake. However, statisticians are not typical users; some modern statistical techniques are so computationally intensive that a billion divisions over a short time period is not outside the realm of possibility. Assuming that the 1 in 9 billion figure is correct and that results of different divisions are independent of one another, what is the probability that at least one error occurs in one billion divisions with this chip?

77. An aircraft seam requires 25 rivets. The seam will have to be reworked if any of these rivets is defective. Suppose rivets are defective independently of one another, each with the same probability.

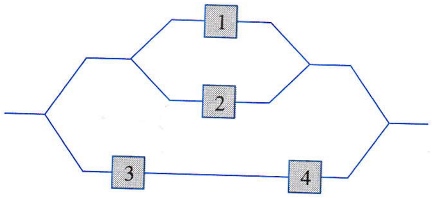
a. If 20% of all seams need reworking, what is the probability that a rivet is defective?

b. How small should the probability of a defective rivet be to ensure that only 10% of all seams need reworking?

78. A boiler has five identical relief valves. The probability that any particular valve will open on demand is .95. Assuming independent operation of the valves, calculate *P*(at least one valve opens) and *P*(at least one valve fails to open).

79. Two pumps connected in parallel fail independently of one another on any given day. The probability that only the older pump will fail is .10, and the probability that only the newer pump will fail is .05. What is the probability that the pumping system will fail on any given day (which happens if both pumps fail)?

80. Consider the system of components connected as in the accompanying picture. Components 1 and 2 are connected in parallel, so that subsystem works iff either 1 or 2 works; since 3 and 4 are connected in series, that subsystem works iff both 3 and 4 work. If components work independently of one another and *P*(component works) = .9, calculate *P*(system works).



81. Refer back to the series-parallel system configuration introduced in Example 2.35, and suppose that there are only two cells rather than three in each parallel subsystem [in Figure 2.14(a), eliminate cells 3 and 6, and renumber cells 4 and 5 as 3 and 4]. Using $P(A_i) = .9$, the probability that system lifetime exceeds t_0 is easily seen to be .9639. To what value would .9 have to be changed in order to increase the system lifetime reliability from .9639 to .99? [*Hint:* Let $P(A_i) = p$, express system reliability in terms of p, and then let $x = p^2$.]

82. Consider independently rolling two fair dice, one red and the other green. Let *A* be the event that the red die shows 3 dots, *B* be the event that the green die shows 4 dots, and *C* be the event that the total number of dots showing on the two dice is 7. Are these events pairwise independent (i.e., are *A* and *B* independent events, are *A* and *C* independent, and are *B* and *C* independent)? Are the three events mutually independent?

83. Components arriving at a distributor are checked for defects by two different inspectors (each component is checked by both inspectors). The first inspector detects 90% of all defectives that are present, and the second inspector does likewise. At least one inspector does not detect a defect on 20% of all defective components. What is the probability that the following occur?

a. A defective component will be detected only by the first inspector? By exactly one of the two inspectors?

b. All three defective components in a batch escape detection by both inspectors (assuming inspections of different components are independent of one another)?

84. Seventy percent of all vehicles examined at a certain emissions inspection station pass the inspection. Assuming that successive vehicles pass or fail independently of one another, calculate the following probabilities:

a. *P*(all of the next three vehicles inspected pass)

b. *P*(at least one of the next three inspected fails)

c. *P*(exactly one of the next three inspected passes)

d. *P*(at most one of the next three vehicles inspected passes)

e. Given that at least one of the next three vehicles passes inspection, what is the probability that all three pass (a conditional probability)?

85. A quality control inspector is inspecting newly produced items for faults. The inspector searches an item for faults in a series of independent fixations, each of a fixed duration. Given that a flaw is actually present, let *p* denote the probability that the flaw is detected during any one fixation (this model is discussed in "Human Performance in Sampling Inspection," *Human Factors,* 1979: 99–105).

a. Assuming that an item has a flaw, what is the probability that it is detected by the end of the second fixation (once a flaw has been detected, the sequence of fixations terminates)?

b. Give an expression for the probability that a flaw will be detected by the end of the *n*th fixation.