Math 3339 Name: SOLUTION

HW #5

You have the following commands in R to compute probabilities associated with Poisson distributions.

`dpois(x, lambda)`: $P(X = x)$ for $X \sim Poisson(\lambda)$
`ppois(q, lambda)`: $P(X \leq q)$ for $X \sim Poisson(\lambda)$

(1)[3 Pts] On average, 2.5 telephone calls per minute are received at the UH's switchboard. Assuming that the number of incoming calls per minute follows a Poisson distribution, compute the probability that at any given minute there will be more than 2 calls.

Denote as X the number of incoming calls per minute. Thus

$$P(X > 2) = 1 - P(X \leq 2)$$

Using R with $\lambda = 2.5$:

```
  > 1-ppois(2,2.5)
[1] 0.4561869
```

(2)[3 Pts] Suppose that in one year the number of industrial accidents X follows a Poisson distribution with mean 3.0. If each accident leads to an insurance claim of $5,000, how much money would an insurance company need to keep in reserve to be 95 % certain that the claims are covered?

You can list the values of the cumulative Poisson distribution with $\lambda = 3$ until you find a value above 0.95. Using R:

```
  > ppois(4,lambda=3)
[1] 0.8152632
  > ppois(5,lambda=3)
[1] 0.9160821
  > ppois(6,lambda=3)
[1] 0.9664915
```

Hence to be 95% confident to be covered, the insurance company should be expected to cover up to 6 claims per year. Thus, it needs to set aside \$ $6 \cdot 5,000$ = \$ 30,000.

(3)[4 Pts] A delivery company found that the number of complaints was six per years on average. Assuming that the number of complaints follows a Poisson distribution, calculate the probability of having no complaints in

(a) all of next year;

(b) the next quarter.

(a) Poisson distribution with $\lambda = 6$. We compute $P(X = 0)$:

```
  > dpois(0,lambda=6)
[1] 0.002478752
```

(b) Poisson distribution with $\lambda = 1.5$. $P(X = 0)$:

```
  > dpois(0,lambda=1.5)
[1] 0.2231302
```

(4)[6 Pts] Let X and Y have the following joint p.d.f.

		x	
y	1	2	3
1	0.05	0.15	0.15
2	0.10	0.10	0.10
3	0.15	0.15	0.05

(a) Calculate the marginal densities. Are X and Y are independent?

(b) Compute the means and variances.

(c) Are X and Y positively correlated? negatively correlated? uncorrelated?

Here is the solution with the R.

```
> p <- matrix(c(.05,.10,.15,.15,.10,.15,.15,.10,.05),ncol=3)
> px <- apply(p,2,sum) ## column-sum:  it creates marginal probabilities for X
> px
[1] 0.3 0.4 0.3
> py <- apply(p,1,sum) ## row-sum:  it creates marginal probabilities for Y
> py
[1] 0.35 0.30 0.35
> x <- c(1,2,3)
> y <- c(1,2,3)
> EX <- sum(px*x)
> EX
[1] 2
> EY <- sum(py*y)
> EY
[1] 2
> EX2 <- sum(px*x*x)
> EY2 <- sum(py*y*y)
> VarX <- EX2-EX*EX
> VarX
[1] 0.6
> VarY <- EY2-EY*EY
> VarY
[1] 0.7
> A=0
> for(i in 1:3)for(j in 1:3)A <- A+p[i,j]*x[i]*y[j]
> EXY<-A
> EXY
[1] 3.8
> COVXY <- EXY-EX*EY
> COVXY
[1] -0.2
```

Negative correlation.

X, Y not independent since $f(2,2) = 0.10 \neq f_1(2) * f_2(2) = 0.4 * 0.3$

(5)[4 Pts] Let $W = 1 - X + 2Y$ be a discrete random variable where X, Y are independent discrete random variables with $\mu_X = 5$, $\mu_Y = 2$, and $\sigma_X^2 = 1$, $\sigma_Y^2 = 2$. Compute μ_W and σ_W^2.

$$\mu_W = 1 - \mu_X + 2\mu_Y = 1 - 5 + (2)(2) = 0$$
$$\sigma_W^2 = \sigma_X^2 + 4\,\sigma_Y^2 = 1 + (4)(2) = 9$$