## HW \#6

Please, write clearly and justify all your steps, to get proper credit for your work.
(1) [6 Pts] Let $X$ and $Y$ have the following joint p.d.f. Compute $\mu_{X}, \mu_{Y}, \sigma_{X}, \sigma_{Y}$ and $\rho$ in each case: (a)

|  | $\mathbf{x}$ |  |  |
| :---: | :---: | :---: | :---: |
| $\mathbf{y}$ | 1 |  | 2 |
| 1 | 0.5 |  | 0 |
| 2 | 0 |  | 0.5 |

(b)

|  | $\mathbf{x}$ |  |  |
| :---: | :---: | :---: | :---: |
| $\mathbf{y}$ | 1 |  | 2 |
| 1 | 0.25 |  | 0.25 |
| 2 | 0.25 |  | 0.25 |

(c)

|  | $\mathbf{x}$ |  |  |
| :---: | :---: | :---: | :---: |
| $\mathbf{y}$ | 1 |  | 2 |
| 1 | 0.1 |  | 0.4 |
| 2 | 0.4 |  | 0.1 |

$(2)[6 \mathrm{Pts}]$ Let $X$ and $Y$ have the following joint p.d.f.

|  |  | $\mathbf{x}$ |  |
| :---: | :---: | :---: | :---: |
| $\mathbf{y}$ | 1 | 2 | 3 |
| 1 | 0.05 | 0.15 | 0.15 |
| 2 | 0.10 | 0.10 | 0.10 |
| 3 | 0.15 | 0.15 | 0.05 |

(a) Calculate the marginal densities. Are $X$ and $Y$ are independent?
(b) Compute the means and variances.
(c) Are $X$ and $Y$ positively correlated? negatively correlated? uncorrelated?
(3) [4 Pts] Let $W=1-X+2 Y$ be a discrete random variable where $X, Y$ are independent discrete random variables with $\mu_{X}=5, \mu_{Y}=2$, and $\sigma_{Y}^{2}=2, \sigma_{X}^{2}=1$. Compute $\mu_{W}$ and $\sigma_{W}^{2}$.
(4) [ 6 Pts$]$ Let $X, Y$ be discrete random variables, where $X=1,2,3,4, Y=1,2,3$, with the joint distribution given by the matrix defined in R below
p <- matrix (c (. $02, .04, .01, .06, .15, .15, .02, .20, .14, .10, .10, .01), \mathrm{ncol}=4)$
Use R to:
(a) Verify that $p$ is a probability mass function (i.e., check that it sums up to 1 )
(a) Define the marginal densities (hint: you can use the apply function) and plot them.
(b) Compute the means and variances.
(c) Are $X$ and $Y$ positively correlated? negatively correlated? uncorrelated?

