Math 3339   Name: SOLUTION
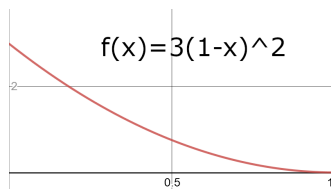
HW #6

To find the numerical solutions of problems 2-4, as presented in class, you can use the statistical tables or the commands `pnorm` and `qnorm` in R.

(1)[4 Pts] let X be a continuous r.v. with pdf $f(x) = 3(1 - x)^2$, $0 \leq x \leq 1$.

(a) Graph the pdf;

f(x)=3(1-x)^2

(b) find the mean of X;
$$\int_0^1 3x(1 - x)^2 dx = 3 \int_0^1 (x - 2x^2 + x^3)dx = 3(\tfrac{x^2}{2} - \tfrac{2x^3}{3} + \tfrac{x^4}{4})\Big|_0^1 = \tfrac{1}{4}$$

(c) compute $P(0.1 < X < 0.5)$;
$$\int_{0.1}^{0.5} 3(1 - x)^2 dx = -(1 - x)^3\Big|_{0.1}^{0.5} = 0.9^3 - 0.5^3 = 0.604$$

(d) compute $P(X > 0.4)$.
$$\int_{0.4}^1 3(1 - x)^2 dx = -(1 - x)^3\Big|_{0.4}^1 = 0.6^3 = 0.216$$

(2)[4 Pts] Let Z be a standard normal random variable and calculate the following probabilities, drawing pictures wherever appropriate

(a) $P(Z \leq 1)$;
$P(Z \leq 1) = \Phi(1) = 0.8413$

(b) $P(|Z| \leq 2.5)$;
$P(|Z| \leq 2.5) = \Phi(2.5) - \Phi(-2.5) = 0.9938 - 0.0062 = 0.9876$

(c) $P(1.37 < Z)$;
$P(1.37 < Z) = 1 - P(Z \leq 1.37) = 1 - \Phi(1.37) = 1 - 0.9147 = 0.0853$

(d) $P(-1.5 < Z < 2)$.
$P(-1.5 < Z < 2) = \Phi(2) - \Phi(-1.5) = 0.9773 - 0.0668 = 0.9105$

(3)[4 Pts] Let X be a normal random variable with mean 12 and standard deviation 3. Calculate the following probabilities

(a) $P(X \leq 4)$;

```
>pnorm(4,mean=12,sd=3)
```
[1] 0.003830381

(a) $P(|X| \le 6)$;

$P(|X| \le 6) = P(-6 < X < 6)$

```
> pnorm(6,mean=12,sd=3)-pnorm(-6,mean=12,sd=3)
```
[1] 0.02275013

(c) $P(X > 4.5)$;

$P(X > 4.5) = 1 - P(X \le 4.5)$

```
> 1-pnorm(4.5,mean=12,sd=3)
```
[1] 0.9937903

(d) $P(-1.5 < X < 4)$

```
> pnorm(4,mean=12,sd=3)-pnorm(-1.5,mean=12,sd=3)
```
[1] 0.003826983

(4)[4 Pts] Determine the value of the constant c that makes the probability statement correct.

(a) $P(0 \le Z \le c) = 0.291$;
$\Phi(c) - \Phi(0) = 0.291 \Rightarrow \Phi(c) = 0.291 + \Phi(0) = 0.791 \Rightarrow c = 0.81$

(a) $P(|Z| \le c) = 0.668$;
$\Phi(c) - \Phi(-c) = 0.668 \Rightarrow 2\Phi(c) - 1 = 0.668 \Rightarrow \Phi(c) = 1.668/2 = 0.834 \Rightarrow c = 0.97$

(c) $P(c < Z) = 0.121$;
$1 - \Phi(c) = 0.121 \Rightarrow \Phi(c) = 0.879 \Rightarrow c = 1.17$

(d) $P(Z < c) = 0.9838$.
$\Phi(c) = 0.9838 \Rightarrow c = 2.14$