HW #7

To find the numerical solutions, you can use the statistical tables or the commands `pnorm` and `qnorm` in R.

(1)[4 Pts] Let $\overline{X}$ be the mean of a random sample of size $n = 48$ from the uniform distribution in the interval $(0, 2)$. Approximate the probability $P(0.9 < \overline{X} < 1.1)$ using the Central Limit Theorem.

By the properties of the uniform distribution, $\mu = 1$, $\sigma^2 = \frac{(0-2)^2}{12} = \frac{1}{3}$
Hence $\mu_{\bar{x}} = 1$, $\sigma^2_{\bar{x}} = \frac{1}{3*48} = \frac{1}{144}$, $\sigma_{\bar{x}} = 1/12$
Using R:
$P(0.9 < \bar{X} < 1.1) =$ `pnorm(1.1,1,1/12)-pnorm(0.9,1,1/12) = 0.7699`

Alternatively, using tables:
$P(0.9 < \bar{X} < 1.1) = P(\frac{0.9-1}{1/12} < Z < \frac{1.1-1}{1/12}) = \Phi(1.2) - \Phi(-1.2) = 0.7698$

(2)[4 Pts] Let $\overline{X}$ be the mean of a random sample of size $n = 48$ from a distribution with mean 4 and variance 16. Approximate the probability $P(3.1 < \overline{X} < 4.6)$ using the Central Limit Theorem.

Note that $\sigma_{\bar{x}} = 4/\sqrt{48}$. *Using R:*
$P(3.1 < \bar{X} < 4.6)$
$=$ `pnorm(4.6,4,4/sqrt(48))-pnorm(3.1,4,4/sqrt(48)) = 0.7911`

Alternatively, using tables:
$P(3.1 < \bar{X} < 4.6) = P(\frac{3.1-4}{4/\sqrt{48}} < Z < \frac{4.6-4}{4/\sqrt{48}}) = \Phi(1.04) - \Phi(-1.56) = 0.792$

(3)[4 Pts] The profits from investments in individual stocks follow a normal distribution with mean 1 and standard deviation 5.

(a) If are buying a single random selected stock, what is the probability that your profit is greater than zero?

(b) If are buying a portfolio of 25 randomly selected stocks, what is the probability that your average profit is greater than zero?

$X \sim N(\mu = 1, \sigma^2 = 25)$
(a) $n = 1$, $\mu_{\bar{X}} = 1$, $\sigma^2_{\bar{X}} = \frac{25}{1}$, $\sigma_{\bar{X}} = 5$
Using R:
$P(\bar{X} > 0) = 1 - P(\bar{X} \leq 0) =$ `1-pnorm(0,1,5) = 0.5792597`
Alternatively, using tables:
$P(\bar{X} > 0) = 1 - P(\bar{X} \leq 0) = 1 - P(Z \leq \frac{0-1}{5}) = 1 - \Phi(-0.2) = 0.5793$
(b) $n = 25$, $\mu_{\bar{X}} = 1$, $\sigma^2_{\bar{X}} = \frac{25}{25}$, $\sigma_{\bar{X}} = 1$
Using R:
$P(\bar{X} > 0) = 1 - P(\bar{X} \leq 0) =$ `1-pnorm(0,1,1) = 0.8413447`
Alternatively, using tables:
$P(\bar{X} > 0) = 1 - P(\bar{X} \leq 0)1 - P(Z \leq \frac{0-1}{1}) = 1 - \Phi(-1) = 0.8413$

(4)[4 Pts] The mean and standard deviation measured from a randomly selected sample of $n = 42$ mathematics SAT test scores are $\overline{x} = 680$ and $s = 35$. Find an approximate 99 percent confidence interval for the population mean μ.

99% confidence interval. $1 - \alpha = 0.99 \Rightarrow \alpha = 0.01 \Rightarrow z_{\alpha/2} = z_{0.005} = 2.576$
Note $z_{0.01/2} =$ `qnorm(1-0.01/2) = 2.575829`

$$[\bar{X} - z_{\alpha/2}\frac{s}{\sqrt{n}}, \bar{X} + z_{\alpha/2}\frac{s}{\sqrt{n}}] = [680 - 2.576\frac{35}{\sqrt{42}}, 680 + 2.576\frac{35}{\sqrt{42}}] = [666.1, 693.1]$$

(5)[4 Pts] A research conducted at the University of Houston wants to estimate the average SAT test scores in mathematics. Assuming that the population of test scores is normally distributed with standard deviation $\sigma = 35$, find the sample size n ensuring that the estimated value of the sample mean is within ± 10 points from the true mean. Use confidence level $\alpha = 0.05$.

Note $z_{0.05/2} =$ `qnorm(1-0.05/2) = 1.959964`
Hence:

$$n \geq \frac{z^2_{\alpha/2}\sigma^2}{h^2} = \left(\frac{1.96 \cdot 35}{10}\right)^2 = 47.1$$

Hence we need at least $n = 48$.