Math 3339 Name:

HW #8

(1)[5 Pts] Let the test statistic W have a t distribution when H_0 is true. Give the significance level for each of the following situations

(i) $H_1 : \mu > m_0$, df=15, rejection region $t \geq 2.947$

(ii) $H_1 : \mu < m_0$, df=24, rejection region $t \leq -2.500$

(iii) $H_1 : \mu \neq m_0$, df=30, rejection region $t \leq -1.697$ or $t \geq 1.697$

(2)[5 Pts] Let μ be the mileage of a certain brand of tire. A sample of $n = 22$ tires is taken at random, resulting in the sample mean $\overline{x} = 29,132$ and sample variance $s^2 = 2,236$.

(a) Assuming that the distribution is normal, find a 99 percent confidence interval for μ.

(b) Repeat the computation of the 99 percent confidence interval for μ is you assume that the variance is known, $\sigma^2 = 2,236$.

(3)[5 Pts] We need to estimate the average of a normal population and from measurements on similar populations we estimate that the sample mean is $s^2 = 9$. Find the sample size n such that we are 90 percent confident that the estimate of $\overline{x}$ is within ± 1 unit of the true mean μ.

(4)[5 Pts] Lightbulbs of a certain type are advertised as having an average lifetime of 750 hours. A random sample of 50 bulbs was selected, the lifetime of each bulb determined finding that the sample average lifetime is 738.5 with sample standard deviation 38.2. Test the hypothesis that the true average lifetime is smaller than what is advertised using significance level $\alpha = 0.01$ and $\alpha = 0.05$.

(5)[5 Pts] A rubber compound were tested for tensile strength and the following values were found 32, 30, 31, 33, 32, 30, 29, 34, 32, 31 Assuming that the population is normally distributed, test the hypothesis that the average tensile strength is different from 31. Use $\alpha = 0.05$. Calculate the p-value of the test.

(6)[5 Pts] The following commands in R computes 5000 simulations of sample means of size 12 from a normal distribution with mean $\mu = 100$ and standard deviation $\sigma = 14$.

```
require(fastR2)
nsamplesum <- do(5000) * c(sample.mean=mean(rnorm(12,100,14)))
```

The following commands compute the approximate mean and standard deviation of the sample mean and plot the histogram giving the approximate distribution of the sample mean.

```
mean(~ sample.mean, data=nsamplesum)
sd(~ sample.mean, data=nsamplesum)
gf_dhistogram(~ sample.mean, data= nsamplesum, bins=20)
```

(a) Compare the approximate values of mean and standard deviation of the sample mean found above with the expected theoretical ones.

(b) Repeat the same simulation as above using now samples from a uniform distribution in the interval $[-2, 4]$. Also in this case, run a numerical test over 5000 simulations, compute mean and standard deviation of the sample mean, and compare it to the theoretical result.