

HW #8

To find the numerical solutions, you can use the statistical tables or the commands `qnorm`, `qt` in R.

(1)[3 Pts] The mean and standard deviation measured from a randomly selected sample of $n = 96$ mathematics SAT test scores are $\bar{x} = 672$ and $s = 31$. Find a 99 percent confidence interval for the population mean μ .

$$1 - \alpha = 0.99 \Rightarrow \alpha = 0.01 \Rightarrow z_{\alpha/2} = z_{0.005} = \text{qnorm}(0.995) = 2.576$$

$$\left[\bar{X} - z_{\alpha/2} \frac{s}{\sqrt{n}}, \bar{X} + z_{\alpha/2} \frac{s}{\sqrt{n}}\right] = \left[672 - 2.576 \frac{31}{\sqrt{96}}, 672 + 2.576 \frac{31}{\sqrt{96}}\right] = [663.9, 680.2]$$

(2)[3 Pts] The mean and standard deviation measured from a randomly selected sample of $n = 16$ mathematics SAT test scores are $\bar{x} = 672$ and $s = 31$. Assume that the scores are normally distributed. Find a 99 percent confidence interval for the population mean μ .

As above, $\alpha = 0.001$. However, here we use $t_{0.005,15} = \text{qt}(0.995, 15) = 2.947$

$$\left[\bar{X} - t_{\alpha/2, n-1} \frac{s}{\sqrt{n}}, \bar{X} + t_{\alpha/2, n-1} \frac{s}{\sqrt{n}}\right] = \left[672 - 2.947 \frac{31}{\sqrt{16}}, 672 + 2.947 \frac{31}{\sqrt{16}}\right] = [649.2, 694.8]$$

(3)[3 Pts] Let a population be normally distributed with mean μ and standard deviation $\sigma = 5$. Find the sample size n such that we are 95 percent confident that the estimate of \bar{x} is within ± 1.5 unit of the true mean μ .

95% confidence interval. $1 - \alpha = 0.95 \Rightarrow \alpha = 0.05 \Rightarrow z_{\alpha/2} = z_{0.025} = 1.96$

$$n \geq z_{0.025}^2 \frac{\sigma^2}{h^2} = 1.96^2 \frac{5^2}{1.5^2} = 42.68$$

Choose $n = 43$

(4)[3 Pts] The EPA considers indoor radon levels above 4 picocuries per liter (pCi/L) of air to be high enough to warrant amelioration efforts. Tests in a sample of 200 homes in a certain county found 127 (63.5%) of these sampled households to have indoor radon levels above 4 pCi/L. Compute the 95% confidence interval of the proportion of all the households in the county that don't meet the EPA guidelines.

95% confidence interval. $1 - \alpha = 0.95 \Rightarrow \alpha = 0.05 \Rightarrow z_{\alpha/2} = z_{0.025} = 1.96$

$$[\bar{X} - z_{\alpha/2} \frac{1}{2\sqrt{n}}, \bar{X} + z_{\alpha/2} \frac{1}{2\sqrt{n}}] = [0.635 - \frac{1.960}{2\sqrt{200}}, 0.635 + \frac{1.960}{2\sqrt{200}}] = [0.566, 0.704]$$

(5)[3 Pts] The EPA considers indoor radon levels above 4 picocuries per liter (pCi/L) of air to be high enough to warrant amelioration efforts. We want to compute the 95% confidence interval of the proportion of all the households in the county that don't meet the EPA guidelines and we want to make sure that our margin of error is within 5 percentage points. How many households do we need to test to guarantee such margin of error

95% confidence interval. $1 - \alpha = 0.95 \Rightarrow \alpha = 0.05 \Rightarrow z_{\alpha/2} = z_{0.025} = 1.96$

Hence we need to choose n such that

$$\frac{z_{\alpha/2}}{2\sqrt{n}} = 0.05, \quad \Rightarrow n = \left(\frac{z_{\alpha/2}}{2 \cdot 0.05} \right)^2 = \left(\frac{1.96}{2 \cdot 0.05} \right)^2 = 384.16$$

Hence we need to choose a sample of 385 households or more.