

Quiz #10

IMPORTANT: In the problems below, you must state clearly (1) the hypothesis you are testing (2) the test statistic you compute and (3) the conclusion of the hypothesis testing problem. You can use R for your computations.

(1)[4 Pts] In comparing the times until failure (in hours) of two different types of light bulbs, we obtain the sample characteristics $n_1 = 38$, $\bar{x} = 964$, $s_x^2 = 6,942$ and $n_2 = 45$, $\bar{y} = 995$, $s_y^2 = 8,330$. Test the hypothesis that the average duration of the second type of light bulbs is higher than the first type at significance level $\alpha = 0.01$.

SOLUTION: We test $H_0 : \mu_1 \geq \mu_2$ against $H_1 : \mu_1 < \mu_2$ with $\alpha = 0.01$.

Test statistic (normal pdf):

$$W = \frac{\bar{x} - \bar{y}}{\sqrt{\frac{s_x^2}{n_1} + \frac{s_y^2}{n_2}}} = \frac{964 - 995}{\sqrt{\frac{6942}{38} + \frac{8330}{45}}} = -1.616436$$

We have that $p\text{-value} = \Phi(-1.616436) = \text{pnorm}(-1.616436) = 0.05300004$

Since the $p\text{-value}$ is larger than $\alpha = 0.01$, then H_0 is NOT REJECTED.

(2)[4 Pts] A study is conducted to evaluate the analgesic effectiveness of a daily dose of oral methadone to reduce pain and each subject is given either methadone or placebo without knowing which treatment they were taking. The table below gives the mean maximum pain intensity scores for each subject, where higher number indicate higher pain. Do data provide sufficient evidence that the maximum pain intensity is lower on days when methadone is taken? Let $\alpha = 0.005$.

Subject	Methadone	Placebo
1	29.8	57.2
2	73.0	69.8
3	98.6	98.2
4	58.8	62.4
5	60.6	67.2
6	57.2	70.6
7	57.9	67.8
8	89.2	95.6

We compare max pain intensity with methadone (m) or placebo (p). We consider the paired differences $d_i = m_i - p_i$

Pair t -test. We test $H_0 : \mu_d \leq 0$ against $H_1 : \mu_d > 0$ with $\alpha = 0.05$.

Data: $n = 11$, $\bar{d} = \frac{1}{11} \sum_{i=1}^{11} d_i = -7.963$, $s_d^2 = 89.857$.

Test statistic (Student t pdf):

$$t = \frac{\bar{d} - \mu_d}{s_{\bar{d}}} = \frac{\bar{d} - \mu_d}{s_d / \sqrt{n}} = \frac{-7.963}{\sqrt{\frac{89.857}{8}}} = -2.376$$

Rejection region: $t < -t_{0.005;7} = -qt(1 - 0.005, 7) = -3.499$

Since $t > -t_{0.005;7}$, then H_0 is NOT REJECTED.

Alternatively:

`t.test(m,p,alternative = "less", paired = TRUE, + var.equal = TRUE, conf.level = 0.995)`

You will find $p\text{-value} = 0.02459$