

Quiz #4

Please recall the following commands in R to compute probabilities associated with binomial, negative binomial and Poisson distributions.

`dbinom(x, size, prob)`: $P(X = x)$ for $X \sim B(\text{size}, \text{prob})$

`pbinom(q, size, prob)`: $P(X \leq q)$ for $X \sim B(\text{size}, \text{prob})$

`dnbinom(x, size, prob)`: $P(X = x)$ for $X \sim NB(\text{size}, \text{prob})$

`pnbinom(q, size, prob)`: $P(X \leq q)$ for $X \sim NB(\text{size}, \text{prob})$

Note that the negative binomial rv in R counts the number of failures that occur before getting the desired success, as I explained in the lectures.

(1) The university football team has 16 games on its schedule. Assume that the probability of winning each game is 0.70 and that there are no ties. Assuming independence, what is the probability that this year's team will have a winning season, that is, that the team will win at least 9 games?

Set $Y \sim b(n = 16, p = 0.7)$. Hence

$$P(Y \geq 9) = 1 - P(Y \leq 8) = 1 - \text{pbinom}(8, 16, 0.7) = 0.9256484$$

(2) If a student answers questions on a true-false test randomly (i.e., assume that $p = 0.5$) and independently, determine the probability that:

- (a) the first correct answer is in response to question 3;
- (b) at most 3 questions must be answered to get the first correct answer.

Set $Z \sim nb(r = 1, p = 0.5)$. Using R where negative binomial counts the number W of failures before the first success:

(a) $P(W = 2) = \text{dnbinom}(2, 1, 0.5) = 0.125$

(b) $P(W \leq 2) = \text{pnbinom}(2, 1, 0.5) = 0.875$