Math 3339 Name:

QUIZ #7

Please, write legibly and show your work. If you use R, you need to report the command you are using. Please, round your solution to 3 decimal digits.

(1)[6 Pts] Let $\overline{X}$ be the mean of a random sample of size $n = 48$ from the uniform distribution in the interval $(2, 8)$. Approximate the probability $P(4.9 < \overline{X} < 5.5)$ using the Central Limit Theorem. You must show how you set up the probability calculation.

By the properties of the uniform distribution, $\mu = \frac{8+2}{2} = 5$, $\sigma^2 = \frac{(8-2)^2}{12} = 3$
Hence $\mu_{\bar{x}} = 5$, $\sigma^2_{\bar{x}} = \frac{3}{48} = \frac{1}{16}$ and $\bar{X} \sim N(\mu = 5, \sigma = 1/4)$

$P(4.9 < \bar{X} < 5.5) = \texttt{pnorm}(5.5, 5, 1/4) - \texttt{pnorm}(4.9, 5, 1/4) = 0.977 - 0.3446 = 0.633$

(2)[4 Pts] Let a population be normally distributed with mean μ and standard deviation $\sigma = 5$. Find the minimal sample size n such that we are 99 percent confident that the estimate of $\overline{x}$ is within ± 1.2 unit of the true mean μ. You must show the formula you apply to find your numerical solution.

$z_{0.005} = \texttt{qnorm}(1 - 0.005) = 2.576$
$n \geq z^2_{0.005} \frac{\sigma^2}{h^2} = 2.576^2 \frac{5^2}{1.2^2} = 115.20$
We choose $n = 116$