

Quiz #8

Please, show your work and write legibly. If you use R, you must report the R command you are using with all relevant parameters. Please, round your results to 1 DECIMAL DIGIT

(1)[10 Pts] Here are the SAT scores of $n = 13$ mathematics SAT test scores:

665, 671, 667, 650, 645, 659, 632, 679, 632, 665, 629, 677, 661

- (a) [2 Pts] Use the R command `mean` to compute the sample mean of the the SAT scores.
 (b) [2 Pts] Use the R command `var` compute the sample variance of the SAT scores.
 (c) [3 Pts] Assuming that the scores are normally distributed, find a 99 percent confidence interval for the population mean μ .
 (d) [3 Pts] Assuming that the scores are normally distributed and that the variance $\sigma^2 = 324$ is known, find the sample size n so that we are 99% confident that the estimate of \bar{x} is within ± 10 unit of the true mean

(a) `> x = c(665, 671, 667, 650, 645, 659, 632, 679, 632, 665, 629, 677, 661)`
`> mean(x) = 656.3`

(b) `> var(x) = 295.7`
`> sqrt(var(x)) = 17.2`

(c) Here $\alpha = 0.01$; Hence $t_{\alpha/2, 12} = t_{0.005, 12} = \text{qt}(0.995, 12) = 3.055$

We use formula

$$\left[\bar{X} - t_{\alpha/2, n-1} \frac{s}{\sqrt{n}}, \bar{X} + t_{\alpha/2, n-1} \frac{s}{\sqrt{n}} \right]$$

Hence

$$\left[656.3 - 3.055 \frac{17.2}{\sqrt{13}}, 656.3 + 3.055 \frac{17.2}{\sqrt{13}} \right] = [641.7, 670.9]$$

(d) For $\alpha = 0.01$, then $z_{\alpha/2} = \text{qnorm}(0.995) = 2.576$

$$n \geq \left(\frac{z_{\alpha/2} \sigma}{h} \right)^2 = \left(\frac{(2.576)(18)}{10} \right)^2 = 21.5 \Rightarrow n = 22$$