

Test #1

Please, write clearly and justify all your steps, to get proper credit for your work. No notes, books or calculators allowed for this test. NOTE: Unless otherwise indicated, the solution should be simplified to an irreducible fraction.

(1)[8 Pts] A bowl contains 20 chips of which 7 are blue chips, 8 are white and 5 are red. Five chips are taken at random drawn and without replacement. Compute the probability that:

- (i) each of the five chips is blue;
- (ii) there are 1 blue, 3 white and 1 red chips;
- (iii) none of the five chips is blue;
- (iv) there is at least 1 blue chip;

NOTE: You can leave the solutions as fractions of binomial coefficients.

(2)[6 Pts] In a box containing 12 light bulbs, 3 are defective. The bulbs are selected at random and tested, one at a time, until the third defective bulb is found. Compute the probability that the third defective bulb is:

- (i) the third bulb tested;
- (ii) the fifth bulb tested;
- (iii) the tenth bulb tested.

(3)[8 Pts] A survey organization asked respondents from 3 different geographical regions what their views were on a certain topic. The answers are reported below.

	East	Midwest	West
Pessimistic	100	90	110
Optimistic	40	70	90
Total	140	160	200

- (i) What is the probability that a randomly selected respondent is pessimistic?
- (ii) What is the conditional probability that a respondent from the Midwest is optimistic?
- (iii) What is the conditional probability that a respondent who is optimistic comes from the Midwest?
- (iv) Are the views of the respondents independent on the geographical regions? Justify your answer. If no, with the same marginal totals, specify what the numbers for the West region would have been, had the two factors been independent.

(4)[4 Pts] A certain disease affects 4% of the population. A test is developed to recognize whether the disease is present, but it does not work perfectly. The probability that the test fails to recognize the presence of the disease is 20%, and the probability of falsely concluding that the disease is present when in fact it is not is 8%.

Calculate the probability that the disease is actually present, when you apply the test and its outcome is positive (i.e., it signals the presence of the disease).

NOTE: You do not need to simplify the numerical solution in this case.