TEST #2

TOT = 22 POINTS

(1) 1 defect in 200 sq. feet $\Rightarrow$ 1.5 defects in 300 sq. feet.

Set $\lambda = 1.5$

(a) let X = # defects in 300 sq. feet. Use Poisson P.D.F, $\lambda = 1.5$

[2Pt] $P(X \le 1) = \sum_{y=0}^{1} \lambda^y \frac{e^{-\lambda}}{y!} = \boxed{0.558}$

(b) $P(X = 0) = e^{-\lambda} = \boxed{0.223}$

[2Pt]

(2) let X = # defective items in one package $X \sim b(n=12, p=0.15)$

[2Pt] (a) $P(X \ge 1) = 1 - P(X=0) = 1 - 0.1422 = \boxed{0.8578}$

[2Pt] (b) $P(X > 1) = 1 - P(X \le 1) = 1 - 0.4435 = \boxed{0.5565}$

[2Pt] (c) Compensation is $2 \cdot X$. Thus $E[2X] = 2\,E[X] = 2 \cdot n \cdot p = \boxed{\$3.60}$

(3) let W = # of defective bulbs. Use hypergeometric distribution

$n = 10 \quad N_1 = 50, \quad N_2 = 950$

[2Pt] (a) $P(W \ge 2) = 1 - P(W \le 1) = 1 - \sum_{w=0}^{1} \frac{\binom{50}{w}\binom{950}{10-w}}{\binom{1000}{10}}$

[2Pt] (b) Use binomial $b(n=10, p=\frac{50}{1000}=0.05)$

$P(W \ge 2) = 1 - P(W \le 1) = 1 - F(1) = 1 - 0.9139 = \boxed{0.0861}$

(4)

Y\X	1	2	
1	0.3	0.1	0.4
2	0.2	0.4	0.6
	0.5	0.5	

X	1	2
$f_1(x)$	0.5	0.5

Y	1	2
$f_2(y)$	0.4	0.6

(a)

[2Pt]

$f(1,1) = 0.3 \ne f_1(1) f_2(1) = 0.5 \cdot 0.4$

This shows that X, Y are DEPENDENT

(b) $\mu_X = 0.5 + 2 \cdot 0.5 = \boxed{1.5}$

$\mu_Y = 0.4 + 2 \cdot 0.6 = \boxed{1.6}$

[4Pt]

$\sigma_X^2 = E[X^2] - (1.5)^2 = 0.5 + 4 \cdot 0.5 - 2.25 = \boxed{0.25}$

$\sigma_Y^2 = E[Y^2] - (1.6)^2 = 0.4 + 4 \cdot 0.6 - 2.56 = \boxed{0.24}$

(c) $E[XY] = 0.3 + 2 \cdot 0.1 + 2 \cdot 0.2 + 4 \cdot 0.4 = 2.5$

$\sigma_{XY} = 2.5 - 1.5 \cdot 1.6 = \boxed{0.1}$

[2Pt]

$$\rho = \frac{\sigma_{XY}}{\sigma_X \sigma_Y} = \boxed{\frac{0.1}{\sqrt{0.25}\,\sqrt{0.24}}} = 0.41$$

X & Y are <u>POSITIVELY CORRELATED</u>