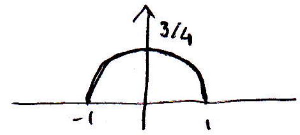
TEST #3

SOLUTION

① $f(x) = \frac{3}{4}(1-x^2) \qquad -1 \le x \le 1$



(a) $E[X] = \int_{-1}^{1} x\, f(x)\, dx = \boxed{0}$ since integral of odd function over symmetric interval

$\sigma^2 = E[(X-\mu)^2] = E[X^2] = \int_{-1}^{1} x^2 f(x)\, dx = 2\int_0^1 \frac{3}{4}(x^2 - x^4)\, dx = \frac{3}{2}\left(\frac{x^3}{3} - \frac{x^5}{5}\right)\Big|_0^1 = \boxed{\frac{1}{5}}$

~~(b) $\int_{-1}^{x_p} f(x)\, dx = 0.5$ Since $\int_{-1}^{0} f(x)\, dx = \int_0^1 f(x)\, dx$, then $x_p = 0$~~

(b) $\int_0^{1/2} f(x)\, dx = \frac{3}{4}\int_0^{1/2}(1-x^2)\, dx = \frac{3}{4}\left(x - \frac{x^3}{3}\right)\Big|_0^{1/2} = \frac{3}{4}\left(\frac{1}{2} - \frac{1}{24}\right) = \frac{33}{48} = \boxed{\frac{11}{32}}$ $= 0.344$

② $n = 27 \qquad \mu_X = \mu_{\bar{X}} = 3 \qquad \sigma_X^2 = \frac{36}{12} = 3 \qquad \sigma_{\bar{X}}^2 = 3/27 = 1/9 \Rightarrow \sigma_{\bar{X}} = 1/3$

$P(2.7 < \bar{X} < 3.2) = P\left(\frac{2.7-3}{1/3} < Z < \frac{3.2-3}{1/3}\right) = P(-0.9 < Z < 0.6) = \Phi(0.6) - \Phi(-0.9)$

$= 0.7257 - 0.1841 = \boxed{0.5416}$

③ $\mu_A = 32, \quad s_A^2 = 1.67, \quad \mu_B = 35, \quad s_B^2 = 2$

NOTE $\quad s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \mu)^2$

$s_p^2 = \frac{6\cdot(1.67) + 5\cdot 2}{11} = 1.82 \qquad s_p = 1.35$

99% confidence interval $\quad t(0.005; r=11) = 3.106$

$(32-35) \pm 3.106 \cdot 1.35\sqrt{\frac{1}{7}+\frac{1}{6}} = -3 \pm 2.331 = [-5.331, -0.669]$

④ $\frac{y_1}{n_1} = \frac{63}{91} = 0.692 \qquad \frac{y_2}{n_2} = \frac{42}{79} = 0.532 \qquad 90\% \Rightarrow \alpha = 0.1 \qquad z(0.05) = 1.645$

INTERVAL: $(0.692 - 0.532) \pm 1.645\sqrt{\frac{0.692\cdot 0.308}{91} + \frac{0.532\cdot 0.468}{79}} = [0.038, 0.282]$

⑤ 95% confidence interval $\quad \alpha = 0.05 \Rightarrow z(0.025) = 1.960. \qquad h = 0.03$

$n = p(1-p)\left[\frac{z(0.025)}{h}\right]^2 = \frac{1}{4}\,\frac{(1.96)^2}{(0.03)^2} = 1067.11$

→ choose $\boxed{n = 1068}$