REVIEW QUIZ #2

2.3-5

x	0	1	2	3
$f(x)$	0.2	0.4	0.3	0.1

$$\mu = \sum_{x=0}^{3} x f(x) = 1.3$$

$$\sigma^2 = E[X^2] - \mu^2 = \sum_{x=0}^{3} x^2 f(x) - (1.3)^2 = 2.5 - 1.69 = 0.81$$

$$\sigma = \sqrt{\sigma^2} = 0.9$$

2.3-6

$$f(x) = \frac{x^2}{30} \qquad x = 1, 2, 3, 4$$

let $u(X) = (4-X)^3$

$$E[u(X)] = E[(4-X)^3] = \sum_{x=1}^{4} (4-x)^3 \frac{x^2}{30} = 3^3 \frac{1}{30} + 2^3 \cdot \frac{4}{30} + \frac{9}{30} = \boxed{\frac{34}{15}} \text{ Ans}$$

2.4-3

$P(\text{High-quality exist}) = 0.10 = p$ (success) $\qquad q = 1 - p = 0.90$

$P(\text{at least one success}) = 1 - P(\text{all failures}) = 1 - q^n$ (n trials)

WANT $\quad 1 - q^n \geq 0.9 \Rightarrow 1 - (0.90)^n \geq 0.9 \Rightarrow 0.1 \geq (0.9)^n$

$$\Rightarrow \boxed{n \geq \frac{\log 0.1}{\log 0.9} = 21.8}$$

2.4-4

$P(\text{roof leaks}) = 0.10 = p$

let Y = # of roofs leaking in 6 buildings $\qquad Y \sim b(n=6, p=0.1)$

(a) $$P(Y \geq 2) = 1 - P(Y \leq 1) = 1 - \sum_{y=0}^{1} \binom{6}{y} (0.1)^y (0.9)^{6-y} = 0.1143$$

$Y \sim b(n=81, p=0.1)$

(b) Money to pay is $U = 100 Y$

$$\mu_U = 100\, \mu_Y = 100 \cdot 81 \cdot 0.1 = 810$$

$$\sigma_U^2 = (100)^2 \sigma_Y^2 = (100)^2\, 81 \cdot 0.1 \cdot 0.9 = 72{,}900 \Rightarrow \sigma_U = 270$$

2.4.8

$P(\text{fish is tagged}) = \frac{4}{10} = p$

Y = # of tagged fishes

(a) $$P(\text{2 of 3 fishes are tagged}) = P(Y=2) = \binom{3}{2} p^2 (1-p)^{3-1}$$

(b) $$P(Y \leq 2) = \sum_{y=0}^{2} \binom{3}{y} p^y (1-p)^{n-y}$$

2.4-9 100 defective items in 2000 items

$n=10$ is taken $\rightarrow$ hypergeometric PDF

(a) W : # of defective items

$$P(W \le 2) = \sum_{k=0}^{2} \frac{\binom{100}{w}\binom{1900}{10-w}}{\binom{2000}{10}} = 0.9887$$

(b) binomial approx $p = \frac{100}{2000} = 0.05$

$$\sum_{w=0}^{2} \binom{10}{w} (0.05)^w (0.95)^{10-w} = 0.9885$$

2.5-1 Poisson approx $\lambda = np = 100 \cdot 0.04 = 4$

(a) $P(X=0) = P(X \le 0) = 0.018$

(b) $P(X=4) = F(4) - F(3) = 0.629 - 0.433 = 0.196$

(c) $P(X > 5) = 1 - F(5) = 1 - 0.785 = 0.215$

2.5-3 $X = \#$ incoming calls $\rightarrow$ Poisson PDF with $\lambda = 5$

$P(X > 10) = 1 - F(10) = 1 - 0.9863 = 0.0137$

2.5-4 $X = \#$ plants shutdown $\rightarrow$ Poisson PDF with $\lambda = 2$

(a) $P(X > 3) = 1 - F(3) = 0.143$

(b) $P(X \ge 1) = 1 - F(0) = 1 - e^{-2} = 0.865$

(c) $E(\text{loss}) = E(1000 X) = 1000 \cdot 2 = 2000$

2.6-9 ~~$x = 1, 2, 3$~~ $f(x) = \frac{1}{3}$ $x = 1, 2, 3$

x_1 \ x_2	1	2	3
1	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$
2			
3	–	–	–

(a) $\mu_X = \frac{1}{3} + \frac{2}{3} + \frac{3}{3} = 2$ $\sigma_X^2 = \frac{1}{3} + \frac{4}{3} + \frac{9}{3} - 4 = \frac{2}{3}$

$Y = X_1 + X_2$ $\mu_Y = \mu_{X_1} + \mu_{X_2} = 4$

$$\sigma_Y^2 = \sigma_{X_1}^2 + \sigma_{X_2}^2 + \underbrace{2\sigma_{X_1 X_2}}_{=0 \text{ since independent}} = \frac{2}{3} + \frac{2}{3} = \frac{4}{3}$$

(b) $Y = X_1 + X_2$ $R = 2, 3, 4, 5, 6$

$P(Y=2) = P(X_1 = 1 \,\&\, X_2 = 1) = \frac{1}{9}$ $P(Y=3) = P((1,2) \text{ or } (2,1)) = \frac{2}{9}$

$P(Y=4) = P((2,2)$ or $(1,3)$ or $(3,1)) = \frac{3}{9}$ $P(Y=5) = P((2,3) \text{ or } (3,2)) = \frac{2}{9}$, $P(Y=6) = P(3,3) = \frac{1}{9}$

$\mu_Y = 2 \cdot \frac{1}{9} + 3 \cdot \frac{2}{9} + 4 \cdot \frac{3}{9} + 5 \cdot \frac{2}{9} + 6 \cdot \frac{1}{9} = 4$

$\sigma_Y = 4/3$