REVIEW for QUIZ #3

SEC 3.1

3.1-4 $f(x) = 3(1-x)^2$

(a) $P(0.1 < X < 0.5) = 3\int_{0.1}^{0.5} (1-x)^2 dx = -(1-x)^3 \Big|_{0.1}^{0.5} = (0.9)^3 - (0.5)^3 = 0.604$

(b) $P(X > 0.4) = 3\int_{0.4}^{1} (1-x)^2 dx = -(1-x)^3 \Big|_{0.4}^{1} = (0.6)^3 = 0.216$

(c) $P(0.3 < X < 2) = 3\int_{0.3}^{1} (1-x)^2 dx = -(1-x)^3 \Big|_{0.3}^{1} = (0.7)^3 = 0.343$

3.1-6 (b) $f(x) = e^{-x} \quad x \geq 0$

$F(x) = \int_0^x e^{-w} dw = 1 - e^{-x} \quad x \geq 0$ Cumulative Dist. F.

Median x: $F(x) = 0.5 \Rightarrow 1 - e^{-x} = 0.5 \Rightarrow -\ln(0.5) = x \Rightarrow x = 0.693$

25th percentile $F(x) = 0.25 \Rightarrow -\ln(0.75) = x \Rightarrow x = 0.288$

75th percentile $F(x) = 0.75 \Rightarrow -\ln(0.25) = x \Rightarrow x = 1.386$

90th percentile $F(x) = 0.90 \Rightarrow -\ln(0.10) = x \Rightarrow x = 2.303$

SEC. 3.2

3.2-3 $N(\mu = 5, \sigma^2 = 4)$

(a) $P(X < c) = 0.8749 \Rightarrow P\left(\frac{X-5}{2} < \frac{c-5}{2}\right) = 0.8749 \Rightarrow P\left(Z < \frac{c-5}{2}\right) = 0.8749$

By table: $\frac{c-5}{2} = 1.15 \Rightarrow \boxed{c = 7.30}$

(b) $P(X > c) = 0.6406 \Rightarrow 1 - P\left(Z < \frac{c-5}{2}\right) = 0.6406 \Rightarrow P\left(Z < \frac{c-5}{2}\right) = 0.3594$

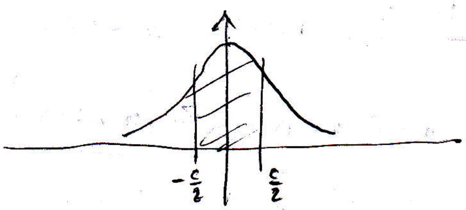
$\Rightarrow \frac{c-5}{2} = -0.36 \Rightarrow \boxed{c = 4.28}$

(c) $\boxed{c = 8.29}$

(d) $P(-c < X - 5 < c) \Rightarrow P\left(-\frac{c}{2} < Z < \frac{c}{2}\right) = 0.95$

$\Rightarrow P\left(Z < -\frac{c}{2}\right) = 0.025 \Rightarrow -\frac{c}{2} = -1.96$

$\Rightarrow \boxed{c = 3.92}$



3.2-4 $\mu = 12.15, \sigma = 0.2 \quad N(12.15, (0.2)^2)$

$P(X < 12) = P\left(Z < \frac{12 - 12.15}{0.2}\right) = \Phi(-0.75) = 0.2266$

Almost 23% of boxes are underweight. This could be increased and variance reduced. Fine is more expensive.

3.2-5. $N(\mu = 0.251, \sigma^2 = (0.001)^2)$

$$P(0.248 \le \bar{X} \le 0.252) = P\left(\frac{0.248-0.251}{0.001} \le Z \le \frac{0.252-0.251}{0.001}\right)$$

$$= P(-3 < Z < 1) = \Phi(1) - \Phi(-3) = 0.84$$

SEC. 3.3

3.3-6 Lognormal pdf with $\mu = 5,\ \sigma = 1$

$$\text{mean} = e^{\mu + \sigma^2/2} = e^{5.5}, \quad \text{VARIANCE} = e^{2\mu+\sigma^2}(e^{\sigma^2}-1) = e^{11}(e-1)$$

$$\text{ST. DEV} = \sqrt{e^{11}(e-1)}$$

$$P(X < 91) = P(\underbrace{\ln X}_{\bar{X} \sim N(5,1)} < \ln 91) = P\left(Z < \frac{\ln 91 - 5}{1}\right) = \Phi(-0.490)$$

$$= \boxed{0.3121}$$

SEC. 3.5

3.5-1 $f(x,y) = 4xy \qquad 0 \le x \le 1, \quad 0 \le y \le 1$

(a) $\displaystyle\int_0^1\int_0^1 4xy = 4\frac{x^2}{2}\Big|_0^1 \frac{y^2}{2}\Big|_0^1 = 1$ ✓

(b) $\displaystyle f_1(x) = \int_0^1 4xy\,dy = 2x \qquad f_2(y) = \int_0^1 4xy\,dx = 2y$

Since $f(x,y) = f_1(x) f_2(y)$, then x, y are INDEPENDENT

SEC 4.1

4.1-1 Uniform distribution $f(x) = \frac{1}{2} \quad x \in [0,2] \qquad \mu_x = 1, \quad \sigma_x^2 = \frac{1}{3}$

$$\mu_{\bar{X}} = 1, \quad \sigma_{\bar{x}}^2 = \frac{1/3}{48} = \frac{1}{144} \qquad (n=48) \qquad \sigma_{\bar{x}} = \frac{1}{12}$$

$$P(0.9 < \bar{X} < 1.1) \approx P\left(\frac{0.9-1}{1/12} < Z < \frac{1.1-1}{1/12}\right) = P(-1.2 < Z < 1.2)$$

$$= \Phi(1.2) - \Phi(-1.2) = 0.8849 - 0.1151 = \boxed{0.7698}$$

4.1-2 $f(x) = \frac{3}{2}x^2, \quad x \in [-1,1] \qquad \mu = \int_{-1}^1 \frac{3}{2}x^3\,dx = \frac{3}{8}x^4\Big|_{-1}^1 = 0 \qquad \sigma^2 = \int_{-1}^1 \frac{3}{2}x^4\,dx = \frac{3}{10}x^5\Big|_{-1}^1 = \frac{3}{5}$

$$\bar{X},\ n = 15 \qquad \mu_{\bar{X}} = 0, \quad \sigma_{\bar{X}}^2 = \frac{3/5}{15} = \frac{1}{25} \implies \sigma_{\bar{X}} = \frac{1}{5}$$

$$P(-0.02 < \bar{X} < 0.1) = P\left(\frac{-0.02}{1/5} < Z < \frac{0.1}{1/5}\right) = \Phi(0.5) - \Phi(-0.1) = 0.6915 - 0.4602 = \boxed{0.2313}$$

4.1-4 PDF : χ^2, $r=6$ $\Rightarrow$ $\mu = 6$, $\sigma^2 = 12$

$\bar{X}$, $n=12$ $\quad \mu_{\bar{X}} = 6$, $\quad \sigma^2_{\bar{X}} = \frac{12}{12} = 1$

$$P(5.1 < \bar{X} < 7.2) = P\left(\frac{5.1-6}{1} < Z < \frac{7.2-6}{1}\right) = \Phi(1.2) - \Phi(-0.9) = \boxed{0.7008}$$

4.1-12 (a) Proportion of smokers $p = \frac{2}{4} = 0.5$

(b) $\binom{4}{2} = \frac{4 \cdot 3}{2} = 6$ random samples of size 2 SS, NN SN, NS

(c) Sampling distribution of proportion $\hat{p}$. outcomes: $\{0, \frac{1}{2}, 1\}$, probability $\{\frac{1}{6}, \frac{4}{6}, \frac{1}{6}\}$

(d) $E(\hat{p}) = 0 \cdot \frac{1}{6} + \frac{1}{2} \cdot \frac{4}{6} + 1 \cdot \frac{1}{6} = 0.5$

This is consistent with central limit theorem

SEC 4.2

4.2-1 $N(\mu, \sigma^2 = 25)$

$\bar{X} = 49.2$, $n = 36$. 90% confidence interval for μ. $\alpha = 0.10$, $\frac{\alpha}{2} = 0.05$

$\hookrightarrow z(0.05) = 1.645$

Interval $\quad 49.2 \pm 1.645 \cdot \frac{5}{\sqrt{36}} = [47.83, 50.57]$

4.2-3 $N(\mu, \sigma^2 = 25)$ 95% confidence interval for μ $\alpha = 0.05$, $\frac{\alpha}{2} = 0.025$

$\hookrightarrow z(0.025) = 1.960$

$$n = \frac{\sigma^2 (z(\frac{\alpha}{2}))^2}{h^2} = \frac{25 \cdot (1.960)^2}{(1.5)^2} = 42.7 \Rightarrow \text{use } \boxed{n = 43}$$

SEC 4.3

4.3-1 (a) $\bar{X} = 2.4$, $\sigma = 0.2$, $n = 22$

95% confidence interval $\rightarrow z(0.025) = 1.960$

INTERVAL $\quad 2.4 \pm 1.960 \cdot \frac{0.2}{\sqrt{22}} = [2.316, 2.484]$

(b) $\bar{X} = 2.4$, $s = 0.2$, $n = 22$

95% confidence interval $\rightarrow t(0.025; n-1=21) = 2.08$

INTERVAL $\quad 2.4 \pm 2.08 \frac{0.2}{\sqrt{22}} = [2.311, 2.489]$

larger interval due to uncertainty in s

4.3-2(a)

$W \sim \chi^2(12)$

$\mu_W = 12, \quad \sigma_W^2 = 24$

4.3-3(a)

$T \sim t(11)$

$\mu_T = 0, \quad \sigma_T^2 = \frac{11}{11-2} = \frac{11}{9} = 1.22$

SEC. 4.4

4.4-4

Set 1 $\quad n_1 = 100, \quad Y_1 = 62 \rightarrow Y_1/n_1 = 0.62$

Set 2 $\quad n_2 = 100, \quad Y_2 = 74 \rightarrow Y_2/n_2 = 0.74$

90% CONFIDENCE INTERVAL for PROPORTIONS $p_1 - p_2$

↳ $z(\frac{\alpha}{2}) = z(0.025) = 1.645$

$$(0.62-0.74) \pm 1.645\sqrt{\frac{(0.62)(0.38)}{100} + \frac{(0.74)(0.26)}{100}} =$$

$$= [-0.23, -0.01]$$

The second group is more successful.

4.4-5

$n=21$ observations for $N(\mu, \sigma^2)$.

$\bar{x} = 74.2, \quad s^2 = 562.8$. 90% confidence interval for σ^2

$\chi^2(0.05; 20) = 31.410, \quad \chi^2(0.95; 20) = 10.851$

INTERVAL: $\left[\frac{(n-1)s^2}{\chi^2(\frac{\alpha}{2}; n-1)}, \frac{(n-1)s^2}{\chi^2(1-\frac{\alpha}{2}; n-1)}\right] = \left[\frac{20 \cdot 562.8}{31.410}, \frac{20 \cdot 562.8}{10.851}\right] = [358.4, 1037.3]$

4.4-7

Want n s.t. 90% confidence interval of $p_1 - p_2$ is within 0.06

WANT $\quad z(0.05)\sqrt{\frac{p_1(1-p_1)}{n} + \frac{p_2(1-p_2)}{n}} \le 0.06 \qquad p_1(1-p_1) \le \frac{1}{4}$

WANT $\quad 1.645\sqrt{\frac{1}{4n} + \frac{1}{4n}} \le 0.06 \Rightarrow n \ge \frac{(1.645)^2}{(0.06)^2}\frac{1}{2} \simeq 376$

Hence we should choose about 376 for each group for a total of 752 samples.