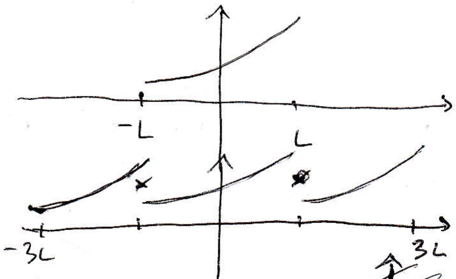
HW #4 SOLUTIONS

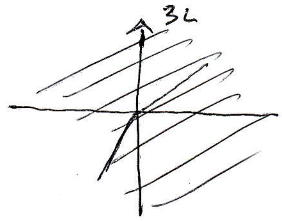
3.2.1 (d) $f(x) = e^x$

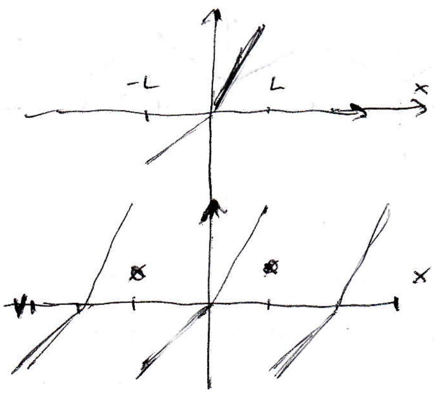


$f(x) = F(f)(x) \quad \forall x \in (-L, L)$

The Fourier series does not converge to f at $x = \pm L$

(e) $f(x) = \begin{cases} x & x < 0 \\ 2x & x > 0 \end{cases}$

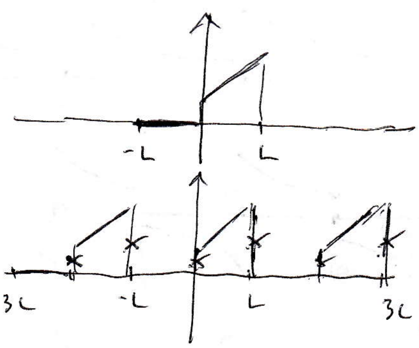




$f(x) = F(f)(x) \quad \forall x \in (-L, L)$

The Fourier series does not converge to f at $x = \pm L$

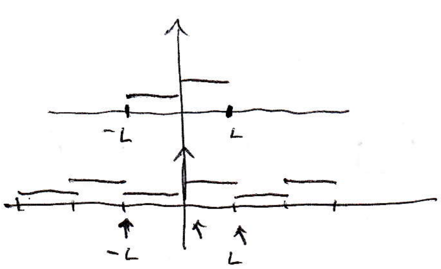
(f) $f(x) = \begin{cases} 0 & x < 0 \\ 1 + x & x > 0 \end{cases}$



The Fourier series does not converge to f at $x = -L, 0, L$

It converges elsewhere on $(-L, 0) \cup (0, L)$

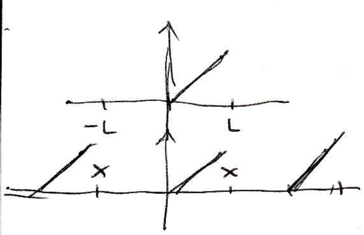
(g) $f(x) = \begin{cases} 1 & x < 0 \\ 2 & x > 0 \end{cases}$



The Fourier series does not converge to f at $x = -L, 0, L$.

It converges to f on $(-L, 0) \cup (L, 0)$

3.2.2 (d) $f(x) = \begin{cases} 0 & x < 0 \\ x & x > 0 \end{cases}$



Fourier series does not converge to f at $x = \pm L$

$$a_0 = \frac{1}{2L} \int_0^L x \, dx = \frac{1}{2L} \cdot \frac{x^2}{2} \Big|_0^L = \frac{L}{4}$$

$$a_n = \frac{1}{L} \int_0^L x \cos \frac{n\pi x}{L} dx = \frac{1}{L} \left[\cancel{\frac{L}{n\pi} x \sin \frac{n\pi x}{L} \Big|_0^L} - \frac{L}{n\pi} \int_0^L \sin \frac{n\pi x}{L} dx \right]$$

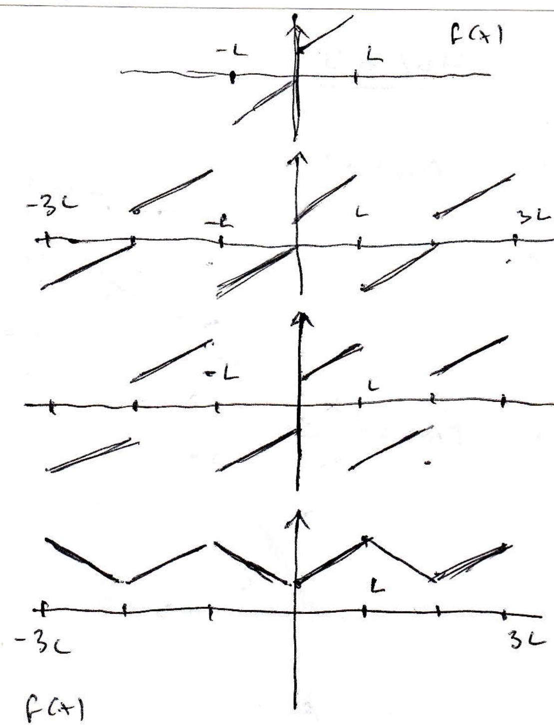
$$= \frac{1}{L} \left(\frac{L^2}{(n\pi)^2} \cos \frac{n\pi x}{L} \Big|_0^L \right) = \frac{L}{(n\pi)^2} \left(\cos n\pi - 1 \right) = \frac{L}{(n\pi)^2} \left((-1)^n - 1 \right)$$

$$b_n = \frac{1}{L} \int_0^L x \sin \frac{n\pi x}{L} dx = \frac{1}{L} \left[-\frac{L}{n\pi} x \cos \frac{n\pi x}{L} \Big|_0^L + \frac{L}{n\pi} \int_0^L \cos \frac{n\pi x}{L} dx \right]$$

$$= \frac{1}{L} \left(-\frac{L^2}{n\pi} \cos n\pi + \cancel{\frac{L^2}{(n\pi)^2} \sin \frac{n\pi x}{L} \Big|_0^L} \right) = -\frac{L}{n\pi} (-1)^n = \frac{L}{n\pi} (-1)^{n+1}$$

$$f(x) \sim a_0 + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi x}{L} + b_n \sin \frac{n\pi x}{L}$$

3.3.1 (c) $f(x) = \begin{cases} x & x<0 \\ 1+x & x>0 \end{cases}$



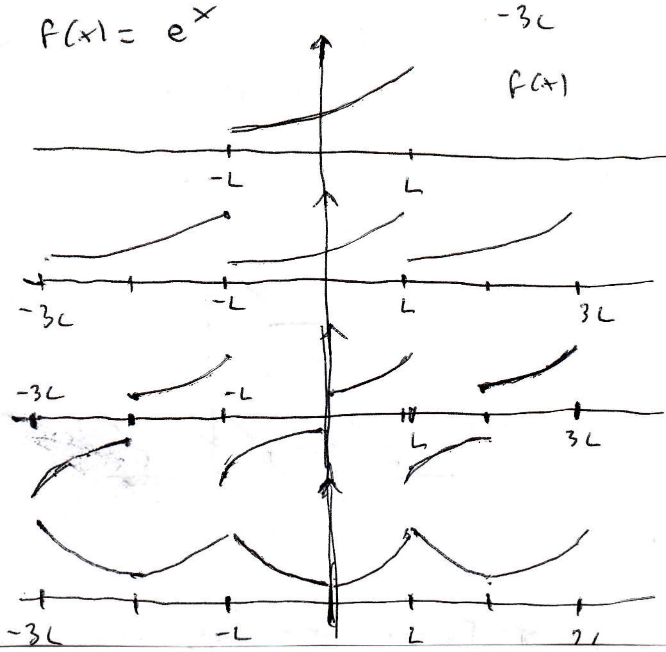
f(x)

Fourier series

SINE SERIES

COSINE SERIES

(d) $f(x) = e^{x}$



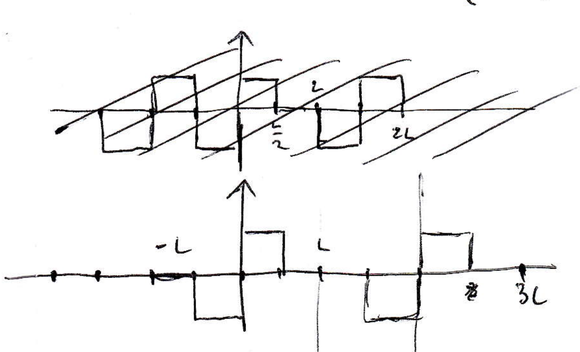
f(x)

FOURIER SERIES

SINE SERIES

COSINE SERIES

3.3.2 (d) $f(x) = \begin{cases} 1 & x<\frac{L}{2} \\ 0 & x>\frac{L}{2} \end{cases}$



Sine series

$$\sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{L}$$

$$b_n = \frac{2}{L}\int_0^{L/2} \sin\frac{n\pi x}{L} = \frac{2}{L}\left(-\frac{L}{n\pi}\cos\frac{n\pi x}{L}\right)\Bigg|_0^{L/2} =$$

$$= \frac{2}{n\pi}\left(1-\cos\frac{n\pi}{2}\right)$$