HW #1

(1) We need to estimate the average porosity of bone samples. From measurements taken on similar populations we estimated that the standard deviation is $\sigma = 0.6$. What sample size n is needed to estimate the true average porosity to within 0.3 with 99% confidence?

$$z_{0.005} = 2.576 \quad n \geq (2.576\,\tfrac{0.6}{0.3})^2 = 26.54 \quad \Rightarrow \boxed{n = 27}$$

(2) The mean caffeine content μ of a certain energy drink is under examination. A measure taken on a random sample of size $n = 16$ yields $\bar{x} = 2.4$ g/l.
  (a) Assuming that the standard deviation is known to be $\sigma = 0.3$, find the 95 confidence interval for μ.
  (b) If that the standard deviation is unknown but the sample standard deviation is $s = 0.3$, find the 95 confidence interval for μ.

(a) $\alpha = 0.05; \quad z_{0.025} = 1.960 \quad$ CI: $\boxed{2.4 \pm 1.960\,\dfrac{0.3}{4} = [2.253, 2.547]}$

(b) $\alpha = 0.05; \quad t(0.025; 15) = 2.131 \quad$ CI: $\boxed{2.4 \pm 2.131\,\dfrac{0.3}{4} = [2.240, 2.560]}$

(3) According to a manufacturer, the average time X taken by a drug to be totally absorbed is 60 min. From measurements on $n = 8$ randomly selected patients, we finds the following data for the absorption times (in minutes):

$$X_1 = 64, X_2 = 59, X_3 = 62, X_4 = 63, X_5 = 60, X_6 = 66, X_7 = 62, X_8 = 61.$$

  (a) Assuming that X is normally distributed, test the hypothesis that the absorption time indicated by the drug manufacturer is too low (that is, test the alternative hypothesis $\mu > 60$), using significance level $\alpha = 0.05$.
  (b) Repeat the test of hypothesis with $\alpha = 0.01$.

$\bar{x} = \tfrac{1}{8}\sum_{i=1}^{8} X_i = \boxed{62.125}; \quad s^2 = \tfrac{1}{7}\sum_{i=1}^{8}(X_i - \bar{x})^2 = 4.982; \quad \boxed{s = 2.232}$
We test $H_0 : \mu = 60$ against $H_1 : \mu > 60$.
  Test statistic (Student t pdf): $t = \dfrac{62.125 - 60}{2.232/\sqrt{8}} = 2.623$
(a) $\alpha = 0.05 \quad t(1 - 0.05; r = 7) = 1.895 \quad \Rightarrow \underline{H_0 \text{ is REJECTED}}$.
(b) $\alpha = 0.01 \quad t(1 - 0.01; r = 7) = 2.998 \quad \Rightarrow \underline{H_0 \text{ is NOT REJECTED}}$.

(4) Minor surgery on horses under field conditions requires a reliable short-term anesthetic producing good muscle relaxation, minimal cardiovascular and respiratory changes, and a quick, smooth recovery with minimal after effects so that horses can be left unattended. The article "A Field Trial of Ketamine Anesthesia in the Horse" (Equine Vet. J.,1984:176–179) reports that for a sample of $n = 75$ horses to which ketamine was administered under certain conditions, the sample average lateral recumbency (lying-down) time was 18.86 min and the standard deviation was 8.6 min. Does this data suggest that true average lateral recumbency time under these conditions is less than 20 min? Test the appropriate hypotheses at level of significance $\alpha = 0.10$ and compute the p value.

We test $H_0 : \mu = 20$ against $H_1 : \mu < 20$ with $\alpha = 0.10$.
Data: $\bar{x} = 18.86$, $\sigma = 8.6$, $n = 75$
Test statistic (Normal pdf): $z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{18.86 - 20}{8.6/\sqrt{75}} = -1.148$
Since $z > z_\alpha = z_{0.10} = -1.282$, then $\underline{H_0 \text{ is NOT REJECTED}}$.
p-value: $P(Z < -1.148) = 0.125$.

(5) A sample of 12 temperature readings (in Fahrenheit) were collected from a bacterial population and resulting readings were as follows:

105.6, 90.9, 91.2, 96.9, 96.5, 91.3, 100.1, 105.0, 99.6, 107.7, 103.3, 92.4.

Does this data suggest that the population mean temperature under these conditions differs from 100? State and test the appropriate hypotheses using $\alpha = 0.05$.

We test $H_0 : \mu = 100$ against $H_1 : \mu \neq 100$ with $\alpha = 0.05$.
Data: $\bar{x} = 98.38$, $\sigma = 6.11$, $n = 12$
Test statistic (Student t pdf): $t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{98.38 - 100}{6.11/\sqrt{12}} = -0.918$
Since $t > t_{\alpha/2;n-1} = -t_{1-0.025;11} = -2.201$, then $\underline{H_0 \text{ is NOT REJECTED}}$.

R solution:

```
> temp <-c(105.6,90.9,91.2,96.9,96.5,91.3,100.1,105.0,99.6,107.7,103.3,92.4)
> t.test(temp,mu=100)


	One Sample t-test

data:  temp
t = -0.92138, df = 11, p-value = 0.3766
```
Since p-value is above 0.05, $\underline{H_0 \text{ is NOT REJECTED}}$.

Quiz #1

A sample of 9 temperature readings (in Fahrenheit) were collected from a bacterial population and resulting readings were as follows:

$$100.1, 96.2, 95.1, 100.5, 96.3, 99.1, 97.1, 100.1, 100.4$$

(a) Compute a 99% confidence interval of the mean of the temperature of the bacterial population.

(b) Assuming that the variance is equal to the sample variance computed in part (a), find the minimal number of samples needed so that the confidence interval of the mean is computed within ± 1 degree Fahrenheit.

(c) Does this data suggest that the population mean temperature under these conditions is less than 100? State and test the appropriate hypotheses using $\alpha = 0.05$.

(d) What is the minimum value of the significance level α at which the null hypothesis can be rejected?

SOLUTION:

A direct calculation gives: $\bar{x} = 98.322$, $s = 2.135$, $n = 9$.

(a) CI: $\bar{x} \pm t_{0.005;8}\frac{s}{\sqrt{n}} = 98.322 \pm \texttt{qt}(1 - 0.005, 8)\frac{2.135}{3} = \boxed{[95.935, 100.709]}$

(b) Need to ensure that $n \geq \left(z_{0.005}\frac{\sigma}{h}\right)^2 = \left(\texttt{qnorm}(1 - 0.005)\frac{2.135}{1}\right)^2 = 30.243$

Thus we need $\boxed{n=31 \text{ samples}}$

(c) We test $H_0 : \mu > 100$ *against* $H_1 : \mu \leq 100$ *with* $\alpha = 0.01$.

Test Statistic (Student t pdf): $T = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{98.322 - 100}{2.135/\sqrt{9}} = -2.358$

Critical Value $= t_{\alpha;n-1} = -\texttt{qt}(1 - 0.05, 8) = -1.860$

Since $T < -1.860$, *then* $\underline{H_0 \text{ is REJECTED}}$.

(d) The minimum value of the significance level α *at which the null hypothesis can be rejected is the p-value:*

p-value $= \texttt{pt(-2.358,8)} = \boxed{0.02305}$

R solution

(a)
```
> x <-c(100.1,95.2,95.1,97.5,96.3,99.7,97.6,98.3,100.4)
> t.test(x,conf.level=0.99)
99 percent confidence interval:
95.93464 100.70980
```
(c)
```
> t.test(x,mu=100,alternative="less")
t = -2.3579, df = 8, p-value = 0.02306
```
$\underline{H_0 \text{ is REJECTED}}$ since p-value below 0.05