Math 4310 – Fall 2023 Name: SOLUTION

HW #2

(7.3.1)

We compare rugby plater (r) vs multisport players (m)

We test $H_0 : \mu_r \leq \mu_m$ against $H_1 : \mu_r > \mu_m$ with $\alpha = 0.01$.

Data: $n_r = 24$, $\bar{x}_r = 27.75$, $s_r = 2.64$; $n_m = 40$, $\bar{x}_m = 22.41$, $s_m = 1.27$. The sample variance is $s_p^2 = \frac{(n_r-1)s_r^2+(n_m-1)s_m^2}{n_r+n_m-2} = 3.600$.

Test statistic (Student t pdf):

$$t = \frac{\bar{x}_r - \bar{x}_m}{\sqrt{\frac{s_p^2}{n_r} + \frac{s_p^2}{n_m}}} = \frac{27.75 - 22.41}{\sqrt{\frac{3.600}{24} + \frac{3.600}{40}}} = 10.90$$

Rejection region: $t > t_{0.01;63} = 2.390$

Since $t > t_{0.005;63} = 2.390$, then H_0 is REJECTED.

NOTE: If one assumes that σ_r and σ_m are known and uses the Normal distribution, then Rejection region: $z > z_{0.01} = 2.326$ and the decision is the same.

(7.3.3)

We compare male patients with obstructive sleep apnea syndrome (o) and healthy male subjects (c).

We test $H_0 : \mu_o = \mu_c$ against $H_1 : \mu_o \neq \mu_c$ with $\alpha = 0.01$.

Data: $n_o = 26$, $\bar{x}_o = 111.060$, $s_o^2 = 48.398$; $n_c = 37$, $\bar{x}_c = 95.854$, $s_c^2 = 31.237$. The sample variance is $s_p^2 = \frac{(n_o-1)s_o^2+(n_c-1)s_c^2}{n_o+n_c-2} = 38.270$.

Test statistic (Student t pdf):

$$t = \frac{\bar{x}_o - \bar{x}_c}{\sqrt{\frac{s_p^2}{n_o} + \frac{s_p^2}{n_c}}} = \frac{111.060 - 95.854}{\sqrt{\frac{38.270}{26} + \frac{38.270}{37}}} = 9.61$$

Rejection region: $t > t_{0.005;62} = 2.660$ or $t < -t_{0.005;62} = -2.660$

Since $t > t_{0.005;63} = 2.660$, then H_0 is REJECTED.

(7.4.3)

We compare max pain intensity with metadone (m) or placebo (p). We consider the paired differences $d_i - p_i - m_i$

Pair t-test. We test $H_0 : \mu_d \leq 0$ against $H_1 : \mu_d > 0$ with $\alpha = 0.05$.

Data: $n = 11$, $\bar{d} = \frac{1}{11}\sum_{i=1}^{11} d_i = 9.618$, $s_d^2 = 102.204$.
Test statistic (Student t pdf):

$$t = \frac{\bar{d} - \mu_d}{s_{\bar{d}}} = \frac{\bar{d} - \mu_d}{s_d/\sqrt{n}} = \frac{9.618}{\sqrt{\frac{102.204}{11}}} = 3.155$$

Rejection region: $t > t_{0.05;10} = 1.812$
Since $t > t_{0.05;10} = 1.812$, then $\underline{H_0 \text{ is REJECTED}}$.

(7.5.1)
We examine proportions of gynecologists-obstetricians in the Flanders who performed at least one cesarean section
We test $H_0 : p \geq 0.35$ against $H_1 : p < 0.35$ with $\alpha = 0.05$.
Data: $n = 295$, $x = 90$, $\hat{p} = \frac{90}{295} = 0.305$.
Test statistic (Standard Normal pdf):

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{0.305 - 0.350}{\sqrt{\frac{(0.35)(0.65)}{295}}} = -1.621$$

Rejection region: $z < -z_{0.95} = -1.645$
Since $z > -z_{0.95} = -1.645$, then $\underline{H_0 \text{ is not REJECTED}}$.

(7.6.2)
We examine rates of posttraumatic stress disorder (PTSD) in mothers (m) and fathers (f).
We test $H_0 : p_f \geq p_m$ against $H_1 : p_f < p_m$ with $\alpha = 0.05$.
Data: $n_f = 175$, $x_f = 28$, $\hat{p}_f = 0.160$; $n_m = 180$, $x_m = 43$, $\hat{p}_m = 0.239$;
Test statistic (Standard Normal pdf):

$$z = \frac{\hat{p}_f - \hat{p}_m}{\hat{\sigma}_{\hat{p}_f - \hat{p}_m}}$$

where

$$\bar{p} = \frac{x_n + x_f}{n_m + n_f} = \frac{71}{355} = 0.200;\ \hat{\sigma}_{\hat{p}_f - \hat{p}_m} = \sqrt{\frac{\bar{p}(1-\bar{p})}{n_f} + \frac{\bar{p}(1-\bar{p})}{n_m}} = 0.042$$

Hence $z = (0.160 - 0.239)/0.042 = -1.667$
Rejection region: $z < -z_{0.95} = -1.645$
Since $z < -z_{0.95} = -1.645$, then $\underline{H_0 \text{ is REJECTED}}$.

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Quiz #2

(1) *Subjects in a Body Mass Index (BMI) study included a sample of 7 male soccer players and a sample of 8 male rugby players whose measured BMI was*

$$\textit{Soccer players: } 23.5, 22.8, 23.2, 23.6, 22.9, 23.1, 23.6$$

$$\textit{Rugby players: } 24.1, 23.8, 24.6, 23.9, 24.9, 24.3, 24.1, 23.6$$

Assume each population sample is normally distributed Is there sufficient evidence for one to claim that, in general, soccer players have a different BMI than rugby players? Assume that the unknown variances are the same and set $\alpha = 0.01$.

Let μ_s and μ_r be the means of the BMI distribution of soccer and rugby players respectively

We test $H_0 : \mu_r = \mu_s$ against $H_1 : \mu_r \neq \mu_s$ with $\alpha = 0.01$.

Using R:

```
> x<-c(23.5, 22.8, 23.2, 23.6, 22.9, 23.1, 23.6)
> y<-c(24.1, 23.8, 24.6, 23.9, 24.9, 24.3, 24.1, 23.6)
> t.test(x,y,alternative = "two.sided", paired = FALSE,var.equal = TRUE)

Two Sample t-test

data:  x and y
t = -4.6047, df = 13, p-value = 0.0004935
alternative hypothesis:  true difference in means is not equal to 0
95 percent confidence interval:
-1.351111 -0.488175
sample estimates:
mean of x mean of y
23.24286 24.16250
```

Since $p - value = 0.0004935 < 0.001$, then $\underline{H_0 \text{ is REJECTED}}$.

(2) *A study compares the rates of posttraumatic stress disorder (PTSD) in mothers (m) and fathers (f). In this study, 28 out of 175 fathers and*

43 out of 180 mothers were found to meet the criteria for PTSD. Is there enough evidence to conclude that fathers are less likely to develop PTSD than mothers. Use $\alpha = 0.01$ and compute the p-value of the hypothesis testing problem.

We test $H_0 : p_f \geq p_m$ against $H_1 : p_f < p_m$ with $\alpha = 0.05$.

Data: $n_f = 175$, $x_f = 28$, $\hat{p}_f = 0.160$; $n_m = 180$, $x_m = 43$, $\hat{p}_m = 0.239$;

Test statistic (Standard Normal pdf):

$$z = \frac{\hat{p}_f - \hat{p}_m}{\hat{\sigma}_{\hat{p}_f - \hat{p}_m}}$$

where

$$\bar{p} = \frac{x_n + x_f}{n_m + n_f} = \frac{71}{355} = 0.200; \ \hat{\sigma}_{\hat{p}_f - \hat{p}_m} = \sqrt{\frac{\bar{p}(1-\bar{p})}{n_f} + \frac{\bar{p}(1-\bar{p})}{n_m}} = 0.042$$

Hence $z = (0.160 - 0.239)/0.042 = -1.667$

Rejection region: $z < -z_{0.99} = -2.326$

Since $z > -z_{0.99} = -1.645$, then $\underline{H_0 \text{ is not REJECTED}}$.

p-value = `pnorm(-1.667) = 0.0477572`

R solution

```
> prop.test(x=c(28,43),n=c(175,180),alternative = "less",conf.level = 0.95, correct
= TRUE)
```

2-sample test for equality of proportions with continuity correction
data: c(28, 43) out of c(175, 180)
X-squared = 2.9759, df = 1, p-value = 0.04226

```
> prop.test(x=c(28,43),n=c(175,180),alternative = "less",conf.level = 0.95, correct
= FALSE)
```

2-sample test for equality of proportions with continuity correction
data: c(28, 43) out of c(175, 180)
X-squared = 3.4514, df = 1, p-value = 0.0316