

HW #1

Please, write clearly and justify all your steps, to get proper credit for your work. You can cite general results from the book or from class.

(1)[6Pts] Let  $x, y \in \mathbb{R}$ .

(a) Prove the triangle inequality:

$$|x + y| \leq |x| + |y|$$

[Hint: You can consider separately the cases where  $x, y$  have different sign or same sign. Or you can derive it from the properties of the absolute values proved in class]

(b) Prove by induction that

$$|x_1 + \cdots + x_n| \leq |x_1| + \cdots + |x_n|$$

(c) Prove the reverse triangle inequality:

$$||x| - |y|| \leq |x - y|$$

(2)[2Pts] Let  $x, a \in \mathbb{R}$ ,  $\epsilon > 0$ . Show that  $|x - a| < \epsilon$  if and only if  $a - \epsilon < x < a + \epsilon$ .

(3)[2Pts] Let  $A, B$  be nonempty subsets of  $\mathbb{R}$ . Let  $A + B = \{a + b : a \in A, b \in B\}$ . Prove that  $\sup(A + B) = \sup A + \sup B$  and that  $\inf(A + B) = \inf A + \inf B$ .

(4)[6Pts] For the following sets, find infimum and supremum and indicate if they have maximum and minimum.

(a)  $A = \{a + a^{-1} : a \in \mathbb{Q}, a > 0\}$

(b)  $B = \{a + (2a)^{-1} : a \in \mathbb{Q}, 0.1 \leq a \leq 5\}$

(c)  $C = \{x e^{-x} : x \in \mathbb{R}\}$

(5)[2Pts] Let  $S = \{1 - (-1)^n/n : n \in \mathbb{N}\}$ . Find  $\sup S$  and  $\inf S$ .