Math 4355 – Spring 2014   Name:

HW #1

Please, write clearly and justify all your steps, to get proper credit for your work.

(1)[5 Pts] Let $v_1 = (v_1, v_2)$ and $u_1 = (u_1, u_2)$ be vectors in $\mathbb{C}^2$ and let
$M = \begin{pmatrix} 2 & -i \\ i & 3 \end{pmatrix}$. Prove that

$$\langle u, v \rangle = (\overline{u_1}, \overline{u_2}) \, M \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$$

defines an inner product on $\mathbb{C}^2$.

(2)[5 Pts] Show that the inner product of $L^2([a, b])$ defined by

$$\langle f, g \rangle = \int_a^b f(t) \, \overline{g(t)} \, dt,$$

is conjugate–symmetric, homogeneous and linear.

This problem is part of Exercise 4 at p. 35 in the textbook. You are encouraged to work out the whole problem on your own, but I will collected the part indicated above. The solution of the positivity part is given in the back of the book.