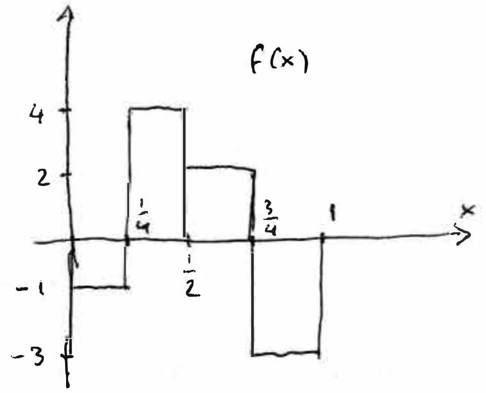
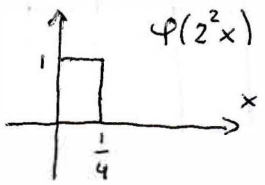
SOLUTION

1





$\{\varphi(2^2x-k) : k\in\mathbb{Z}\}$ basis for V_2

From the picture, it is clear that

$$\boxed{f(x) = -\varphi(2^2x) + 4\varphi(2^2x-1) + 2\varphi(2^2x-2) - 3\varphi(2^2x-3)}$$

$$a^2 = (-1, 4, 2, -3) \to a^1 = \left(\frac{3}{2}, -\frac{1}{2}\right) \to a^0 = (1/2)$$

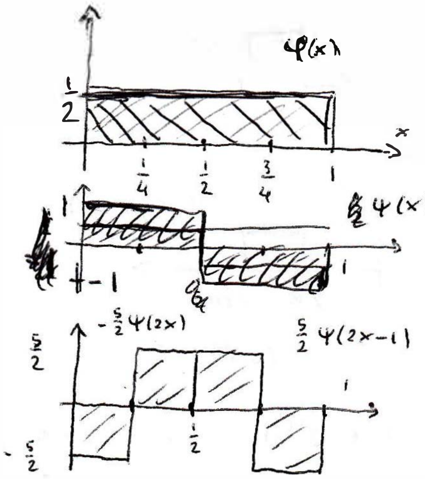
$$\to b^1 = \left(-\frac{5}{2}, \frac{5}{2}\right) \to b^0 = (1)$$

HAAR DECOMPOSITION

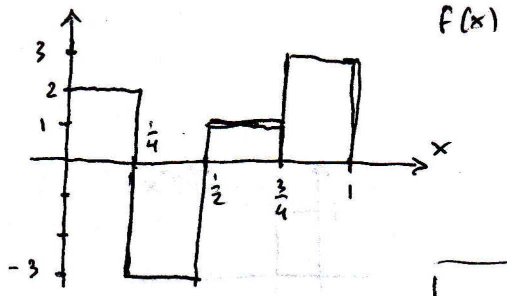
$$a_k^{j-1} = \frac{a_{2k}^j + a_{2k+1}^j}{2}$$

$$b_k^{j-1} = \frac{a_{2k}^j - a_{2k+1}^j}{2}$$

$$\boxed{f(x) = \underset{\downarrow\, V_0}{\frac{1}{2}\varphi(x)} + \underset{\downarrow\, W_0}{1\,\psi(x)} \underbrace{- \frac{5}{2}\psi(2x) + \frac{5}{2}\psi(2x-1)}_{W_1}}$$



②



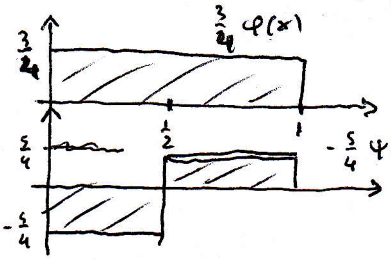
$$\boxed{f(x) = 2\varphi(2x) - 3\varphi(2x-1) + \varphi(2x-2) + 3\varphi(2x-3)}$$

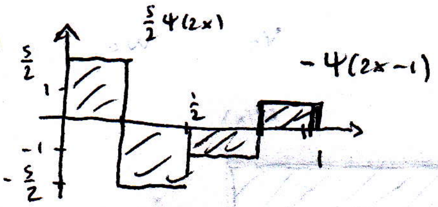
Represetatn with respect to V_2- basis

HAAR DECOMPOSITION

$$\begin{tikzcd} a^2 = (2,-3,1,3) \arrow[r] \arrow[dr] & a^1 = (-\frac{1}{2}, 2) \arrow[r] \arrow[dr] & a^0 = \frac{3}{4} \\ & b^1 = (\frac{5}{2}, -1) & b^0 = -\frac{5}{4} \end{tikzcd}$$

$$\boxed{f(x) = \underbrace{\tfrac{3}{4}\varphi(x)}_{V_0} - \underbrace{\tfrac{5}{4}\psi(x)}_{W_0} + \underbrace{\tfrac{5}{2}\psi(2x) - \psi(2x-1)}_{W_1}}$$





$\varphi(2^n x - n)$

Discrete Haar transform

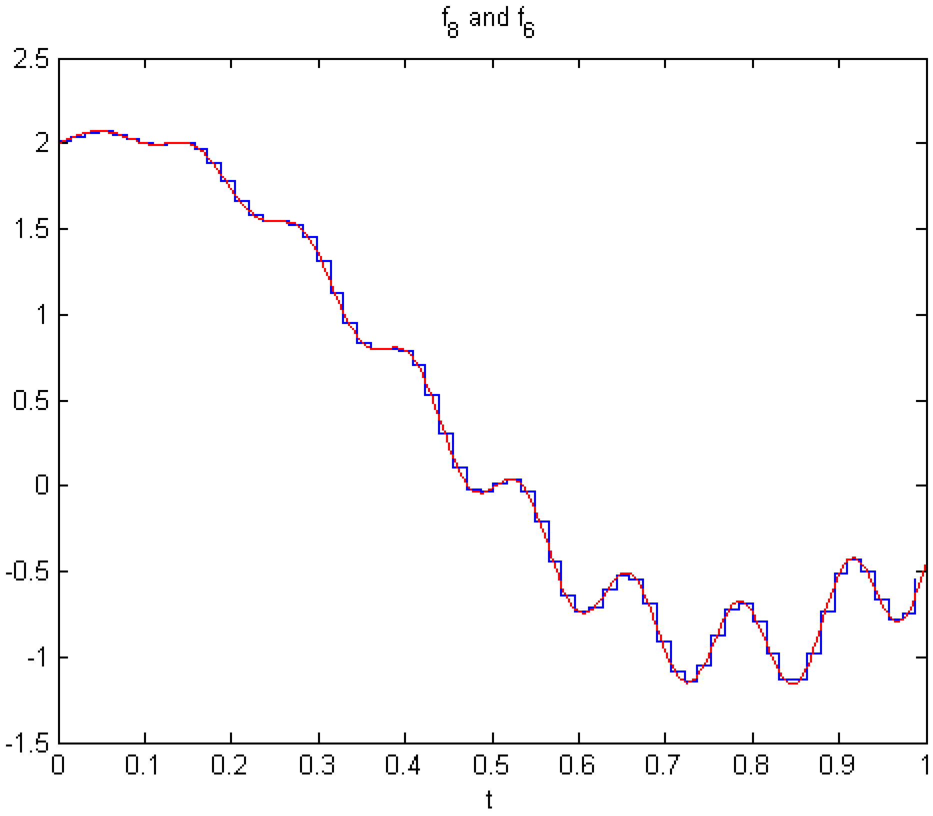
```
function y = mydht(x,j)
% MATLAB function for Haar 1-D decomposition
% This function receives N (a power of two) real values in x
% and the number of j stages (scales) of decomposition
% computes the Haar DWT and returns the N DWT
% coefficients in y.
leng = length(x);  % length of the input vector
b = leng;
  for ns =1:j % outer loop stepping over stages
    p = 1;q = 1;
    while(p < b) % inner loop stepping over each filtering
in a stage
      r = p + 1;
      xmr(q) = (x(p) - x(r))/2; xpr(q) = (x(p) + x(r))/2;
      p = p + 2; q = q + 1;
    end
    x(1:b) = [xpr(1:b/2) xmr(1:b/2)];
    b = b / 2;
  end
y=x;
end
```

Discrete inverse Haar transform

```
function y = myidht(z,j)
% MATLAB function for Haar reconstruction
% This function receives N (a power of two) real values in z
% and the number j of stages (scales) of decomposition,
% computes the Haar IDWT, and returns the N real values in y.
N = length(z); % length of the input vector
b = N/2^j;
z(1:b) = z(1:b);
  for ns =1:j  % outer loop stepping over stages
    p = 1;q = 1;
    xpr(1:b) = z(1:b); xmr(1:b) = z(b+1:2*b);
    while(p <= b)  % inner loop stepping over each filter in a
stage
      z(q + 1) = xpr(p) - xmr(p); z(q) = xpr(p) + xmr(p);
      p = p + 1; q = q + 2;
    end
    b = b * 2;
  end
  y = z;
end
```

Script to plot Haar decomposition

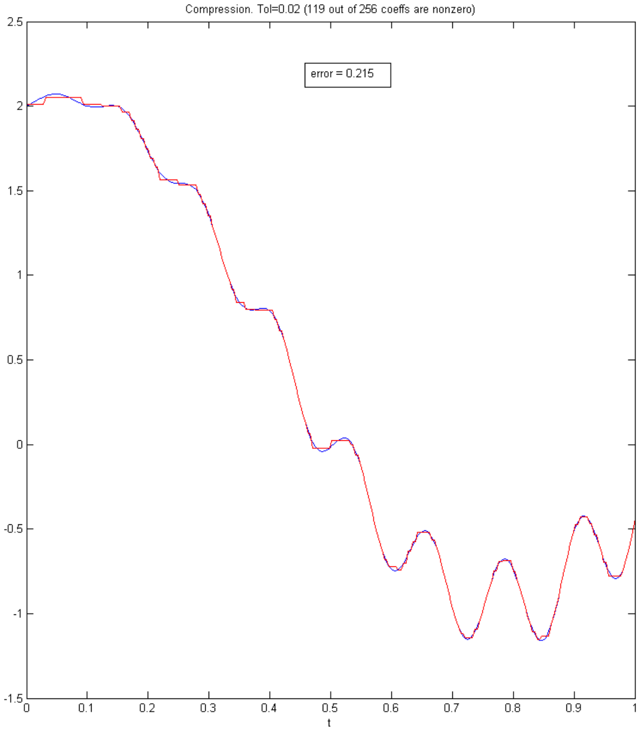
```
>> t = 0:1/(2^(8)-1):1;
>> f = exp(-t.^2/10).*(sin(2*t) + 2*cos(4*t) + 0.4*sin(t).*sin(50*t));
>> t3 = 0:4/(2^(8)-1):1;
>> y=mydht(f,2);
>> stairs(t3,y(1:64))
>> hold on
>> plot(t,f,'r')
```

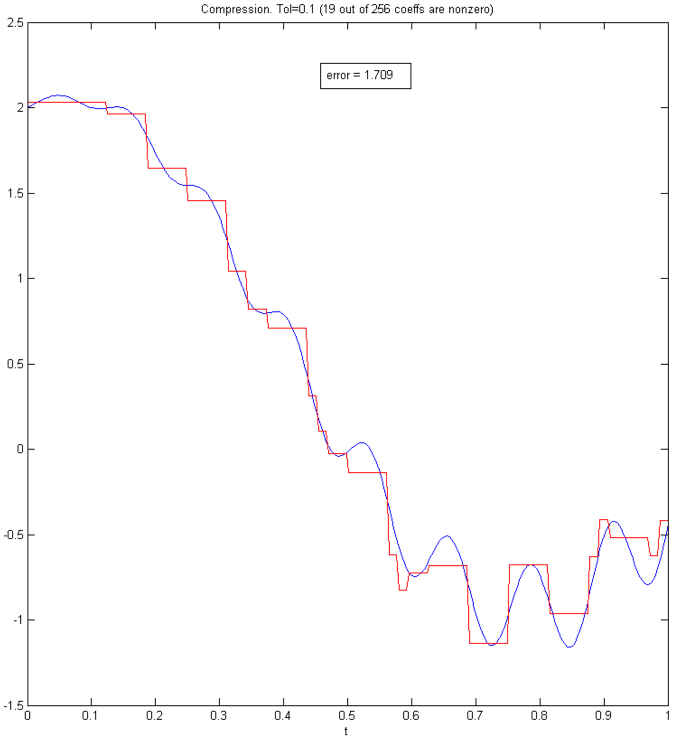


Compression using HWT

```
>> y=mydht(f,8);
>> yc=y.*(abs(y)>0.02);
>> fc=myidht(yc,8);
>> plot(t,f,t,fc,'r')
>> norm(f-fc)
ans =
    0.2150
>> nnz(yc)
ans =
   119
```

```
>> y=mydht(f,8);
>> yc=y.*(abs(y)>0.1);
>> fc=myidht(yc,8);
>> plot(t,f,t,fc,'r')
>> norm(f-fc)
ans =
    1.7088
>> nnz(yc)
ans =
    19
```


Compression. Tol=0.02 (119 out of 256 coeffs are nonzero)
error = 0.215
t


Compression. Tol=0.1 (19 out of 256 coeffs are nonzero)
error = 1.709
t