Math 4355 – Spring 2021 Name:

HW #1

(1) Let $v = (v_1, v_2)$ and $u = (u_1, u_2)$ be vectors in $\mathbb{C}^2$ and let $M = \begin{pmatrix} 2 & -i \\ i & 3 \end{pmatrix}$. Prove that

$$\langle u, v \rangle = (\overline{v_1}, \overline{v_2})\, M \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

defines an inner product on $\mathbb{C}^2$ by showing that each of the 4 properties of the inner product is satisfied

(2) (a) Show that the inner product of $L^2([a, b])$ defined by

$$\langle f, g \rangle = \int_a^b f(t)\, \overline{g(t)}\, dt,$$

is conjugate–symmetric, homogeneous and linear.

(b) Show that the inner product defined above satisfies the positivity property.

This problem is part of Exercise 4 at p. 35 in the textbook. You are encouraged to work out the problem on your own by following the hints. The solution of the positivity part is given in the back of the book.