HW2 SOLUTION

(7) [3Pts]

$$\|f_n - 0\|^2 = \int_0^{\frac{1}{n^2}} (\sqrt{n})^2 dt = \frac{2}{3}\frac{1}{n}$$

$$\lim_{n\to\infty} \|f_n - 0\| = \lim_{n \to 0} \sqrt{\frac{2}{3}\frac{1}{n}} = 0$$

This shows that $f_n \to 0$ in $L^2([0,1])$

However, $f_n(0) = \sqrt{n} \quad \forall n$

$$\lim_{n\to\infty} f_n(0) = \infty.$$

This shows that f_n does not converge to $f=0$. Pointwise

(9) [3Pts]

Let $V = \text{span}\{(1,-2,1)\}$.

$$V^\perp = \{(x_1,x_2,x_3) : \langle (x_1,x_2,x_3), (1,-2,1)\rangle = 0\}$$

$$= \{(x_1,x_2,x_3) : x_1 - 2x_2 + x_3 = 0\}$$

$V^\perp$ corresponds to the plane whose normal vector is $(1,-2,1)$

(10) [2Pts]

Let $f(t) = 1$, $t \in [0,1]$

$$\{h \in L^2[0,1] : \langle f, h\rangle = 0\} = \{h \in L^2[0,1] : \int h(t)\, dt = 0\}$$

Hence, the orthogonal COMPLEMENT of f in $L^2[0,1]$ are the functions in $L^2[0,1]$ whose average is zero.