Math 4355 – Fall 2015 Name:

HW #3

Please, write clearly and justify all your steps, to get proper credit for your work.

(1)[8 Pts] Let $V = L^2([-\pi, \pi])$ and consider the subspace $V_0 \subset V$ given by

$$V_0 = \text{span}\{1, \cos x, \sin x\}$$

(i) Find an ON basis for V_0 (note that V_0 is a subspace of $L^2([-\pi, \pi])$ so that the functions are only defined on $[-\pi, \pi]$).

(ii) Show that the space $V_1 = \text{span}\{\cos(2x), \sin(2x)\} \subset V$ is orthogonal to V_0. Is V_1 the orthogonal complement of V_0? Justify your answer.

(iii) Find the orthogonal projection of $f(x) = \cos(3x)$ onto V_0

(iv) Find the orthogonal projection of $f(x) = x$, for the interval $[-\pi, \pi]$, onto V_0 and onto V_1.

(2)[3 Pts] Consider the inner product space $V = L^2([0, 1])$. Compute the orthogonal projection of the function $f(x) = e^{-x}$, for $x \in [0, 1]$, onto the subspace $V_0 = \text{span}\{\phi, \psi\}$, where

$$\phi(x) = \begin{cases} 1 & 0 \le x < 1 \\ 0 & \text{otherwise.} \end{cases}$$

$$\psi(x) = \begin{cases} 1 & 0 \le x < \frac{1}{2} \\ -1 & \frac{1}{2} \le x < 1 \\ 0 & \text{otherwise.} \end{cases}$$