Math 4355 – Spring 2014 Name:

HW #4

(1) [8 Pts] Solve Ex 1,8, p.83-84.

(2) [4 Pts] This problem is about *numerical* approximation of functions using Fourier series.

Let $f(x) = x$, for $x \in [-\pi, \pi]$. We computed in class its Fourier series, which is also a sine series (that is, the cosine terms vanish):

$$F(x) = \sum_{k=1}^{\infty} (-1)^{k+1} \frac{2}{k} \sin(kx)$$

Denote the partial sums as

$$F_N(x) = \sum_{k=1}^{\mathbb{N}} (-1)^{k+1} \frac{2}{k} \sin(kx)$$

(i) Use Matlab to produce the plots of $F_3(x)$, $F_5(x)$, $F_9(x)$. Each time, compare the plot with $f(x)$, that is, show both $F_N(x)$ and $f(x)$ on the same plot. Please, choose a window of size $[-6, 6]$, so that the behavior of the approximations at the endpoints $\pm\pi$ will be visible.

(ii) You will notice some blips in the approximating functions, near the locations $\pm\pi$. This is called the *Gibbs phenomenon.* Estimate from your graphs the height of the maximum height of the blips for $F_3(x)$, $F_5(x)$, $F_9(x)$.

NOTICE: Graphs have to be properly labelled. While it is strongly encouraged to use Matlab, you are allowed to also use Octave, Scilab, or Mathematica, if it is impractical for you to use Matlab.