HW4 SOLUTION

(1) → **Ex. 1** Expand $f(x) = x^2$ in a Fourier series valid on $[-\pi, \pi]$

Note: since $f(x)$ is even, then $b_k = \int_{-\pi}^{\pi} f(x) \sin kx \, dx = 0 \quad \forall k$

(3Pts)

$$a_0 = \frac{1}{\pi} \int_0^{\pi} x^2 \, dx = \frac{1}{\pi} \frac{\pi^3}{3} = \frac{\pi^2}{3}$$

$$a_k = \frac{2}{\pi} \int_0^{\pi} x^2 \cos kx = \frac{2}{\pi} \left(\cancel{\frac{x^2}{k} \sin kx \Big|_0^{\pi}} - \int_0^{\pi} \frac{2x}{k} \sin kx \, dx \right)$$

$$= \frac{2}{\pi} \left(\frac{2x}{k} \cos kx \Big|_0^{\pi} + \frac{2}{k^2} \cancel{\int_0^{\pi} \cos kx \, dx} \right) = \frac{4}{k^2} (-1)^k$$

Thus

$$\boxed{f(x) \sim \frac{\pi^2}{3} + \sum_{k=1}^{\infty} \frac{4}{k^2} (-1)^k \cos kx}$$

PLOTS → [1Pt] Matlab plot for $N = 1, 2, 5, 7$

Ex. 8 Expand $f(x) = \begin{cases} 1 & x \in [-1/2, 1/2] \\ 0 & x \in [-1, 1] \setminus [-1/2, 1/2] \end{cases}$ in a Fourier series valid in $[-1, 1]$

(3Pts)

Note: since $f(x)$ is even, then $b_k = 0 \quad \forall k$

$$a_0 = \int_0^1 f(x) \, dx = \int_0^{1/2} dx = 1/2$$

$$a_k = 2 \int_0^1 f(x) \cos(k\pi x) \, dx = 2 \int_0^{1/2} \cos(k\pi x) \, dx = \frac{2}{k\pi} \sin\left(k \frac{\pi}{2}\right)$$

Set $k = 2m$ $\quad a_{2m} = \frac{1}{m\pi} \sin(m\pi) = 0$

$k = 2m-1$ $\quad a_{2m-1} = \frac{2}{(2m-1)\pi} \sin\left((2m-1)\frac{\pi}{2}\right) = \frac{2}{(2m-1)\pi} (-1)^{m+1}$

Thus

$$\boxed{f(x) \sim \frac{1}{2} + \sum_{k=1}^{\infty} \frac{2}{(2m-1)\pi} (-1)^{m+1} \cos((2m-1)\pi x)}$$

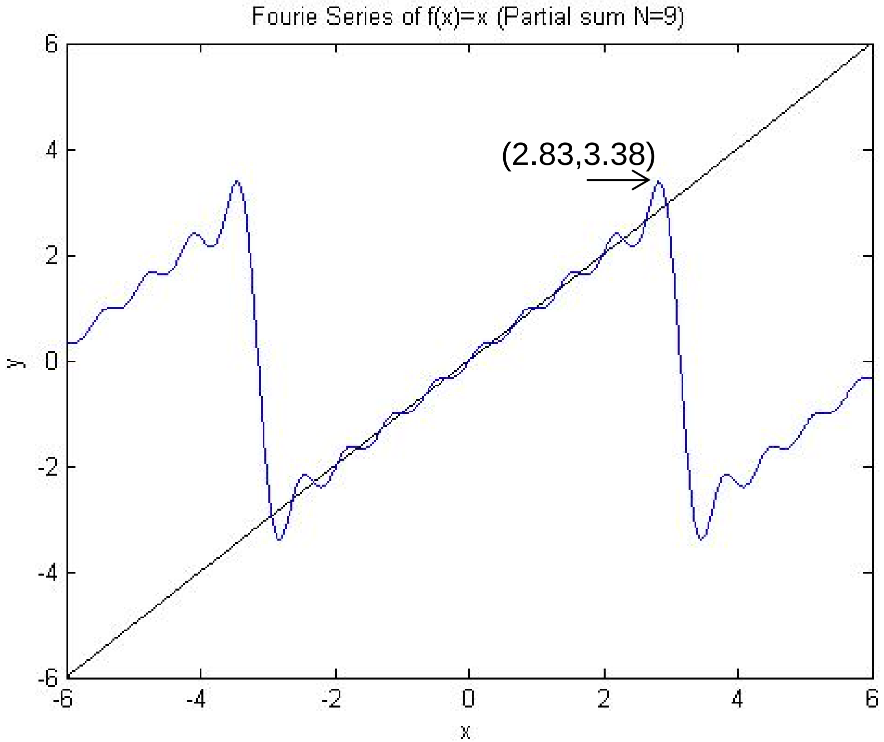
PLOTS → [1Pt] Matlab plot for $N = 5, 10, 20, 40$

(2) PLOTS → [4 Pts] (i) Matlab plots for $N = 3, 5, 9$ (3Pts)

(ii) Estimate Gibbs phenomenon (1Pt)

Fouries series of f(x) =x, [-π,π]

```
x=[-6:0.01:6];
f=x;
F=0;
for k=1:9
F=(-1)^(k+1)*(2*sin(k*x)/k)+F;
end
plot(x,f,'k',x,F,'b')
title('Fourie Series of f(x)=x (Partial sum N=9)')
xlabel('x')
ylabel('y')
```



Overshoot = 0.55