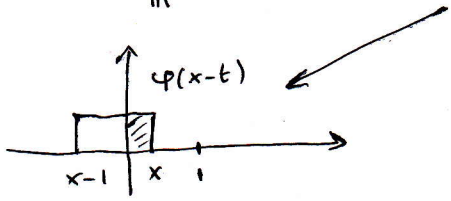
HW 8 SOLUTION

(5) $\varphi(t) = \chi_{[0,1]}(t)$

$$\varphi * \varphi(x) = \int_{\mathbb{R}} \varphi(t)\,\varphi(x-t)\,dt = \int_0^1 \varphi(x-t)\,dt = \begin{cases} 0 & \text{if } x \geq 2 \\ 0 & \text{if } x < 0 \\ x & \text{if } 0 \leq x < 1 \\ 2-x & \text{if } 1 \leq x < 2 \end{cases}$$



$$= \boxed{\begin{cases} 1-|x-1| & \text{if } 0 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}}$$

(10) $h(t) = \begin{cases} A e^{-\alpha t} & t \geq 0 \\ 0 & \text{otherwise} \end{cases}$

$$\hat{h}(\omega) = \frac{1}{\sqrt{2\pi}} \int_{\mathbb{R}} h(t)\, e^{-i\omega t}\, dt = \frac{A}{\sqrt{2\pi}} \int_0^\infty e^{-(\alpha+i\omega)t}\, dt = \frac{A}{\sqrt{2\pi}} \frac{-e^{-(\alpha+i\omega)t}}{\alpha+i\omega}\Big|_0^\infty$$

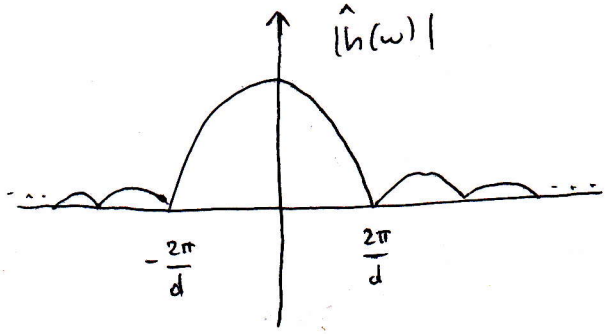
$$= \boxed{\frac{A}{\sqrt{2\pi}\,(\alpha+i\omega)}}$$

(12) $h(t) = \frac{1}{d}\chi_{[0,d]}(t)$ $\qquad \hat{h}(\omega) = \frac{1}{\sqrt{2\pi}} \frac{1}{d} \int_0^d e^{-i\omega t}\, dt = \frac{i}{\sqrt{2\pi}\, d\omega}\left(e^{-i\omega d} - 1\right)$

$$= \frac{1}{\sqrt{2\pi}\, d\omega}\left(\sin(\omega d) + i(\cos\omega d - 1)\right)$$

$$|\hat{h}(\omega)| = \frac{1}{\sqrt{2\pi}\, d|\omega|} \sqrt{\sin^2(\omega d) + (\cos(\omega d)-1)^2} = \frac{1}{\sqrt{\pi}} \frac{\sqrt{1-\cos(\omega d)}}{|\omega| d}$$

$$= \frac{\sqrt{2}\,|\sin(\omega \frac{d}{2})|}{\sqrt{\pi}\,|\omega| d} = \frac{1}{\sqrt{2\pi}} \left|\operatorname{sinc}\left(\frac{\omega d}{2}\right)\right|$$



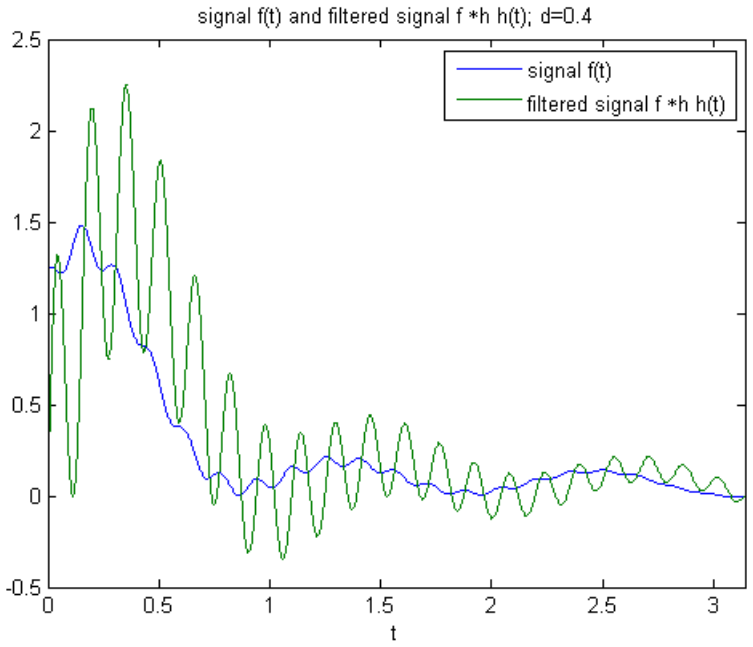
First zero at $\frac{\omega d}{2} = \pi \Rightarrow \omega = \frac{2\pi}{d}$

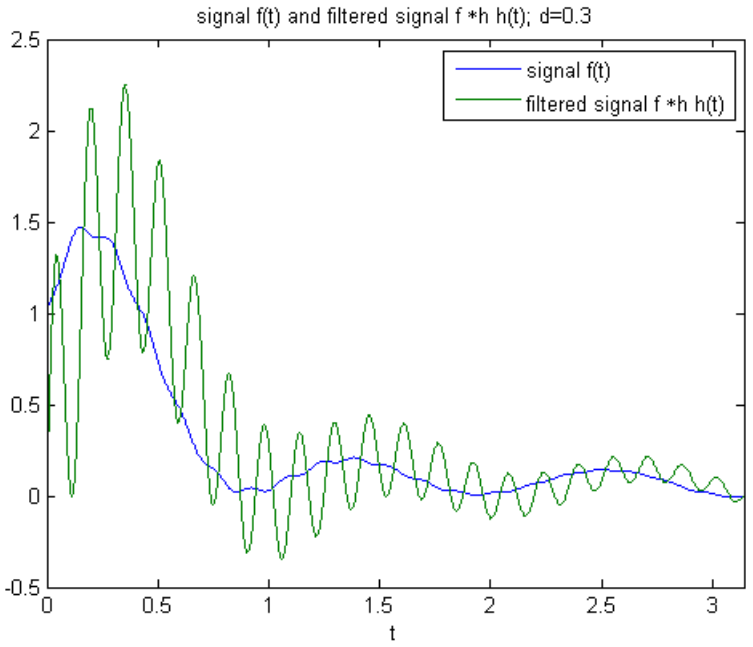
To retain Frequencies ≤ 5, set

$$5 \leq \frac{\pi}{d} \Rightarrow \boxed{d \leq \frac{\pi}{5} \simeq 0.6}$$

To remove Frequencies ≥ 30 set

$$30 \geq \frac{2\pi}{d} \Rightarrow \boxed{d \geq \frac{2\pi}{30} \simeq 0.2}$$





MATLAB CODE

```
% Define time range
t=0:0.0001:pi;
% Define d
d=0.4;
% Define f
f = exp(-t).*(sin(t)+sin(3*t)+sin(5*t)+sin(40*t));
% Define h. Note that its area should be one.
% To achieve this, I divide by its length.
norm=sum(t>=0 & t<=d);
hd = (t>=0 & t<=d)./norm+(t>d).*0;
ffiltered = conv(f,hd);
% The convolved signal has support longer than the support of f.
% I choose a window corresponding to the locations where the filter
% support is contained inside the support of f
len=length(t);
plot(t,ffiltered(norm:len+norm-1),t,f)
title('signal f(t) and filtered signal f \asth h(t); d=0.4');
axis([0 pi -0.5 2.5]);xlabel('t');
legend('signal f(t)','filtered signal f\asth h(t)')
```