Math 4355 – Spring 2021 Name:

Quiz #1

Please, type or write legibly, scan, save file as LASTNAME_FIRSTNAME_Q1.pdf and email to `dlabate@math.uh.edu` or `dlabate@uh.edu`. You need to email to me no later than 11:30AM on Jan 28.

Let $v = (v_1, v_2)$ and $u = (u_1, u_2)$ be vectors in $\mathbb{C}^2$ and let $M = \begin{pmatrix} a & i \\ -i & b \end{pmatrix}$ where a, b are fixed real numbers Prove that

$$\langle u, v \rangle = (\overline{v_1}, \overline{v_2})\, M \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

is conjugate symmetric, that is, $\langle u, v \rangle = \overline{\langle v, u \rangle}$

Proof.

$$\begin{aligned} \langle u, v \rangle &= (\overline{v_1}, \overline{v_2})\, M \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \\ &= (\overline{v_1}, \overline{v_2}) \begin{pmatrix} au_1 + iu_2 \\ -iu_1 + bu_2 \end{pmatrix} \\ &= au_1\overline{v_1} + iu_2\overline{v_1} - iu_1\overline{v_2} + bu_2\overline{v_2} \end{aligned}$$

$$\begin{aligned} \langle v, u \rangle &= (\overline{u_1}, \overline{u_2})\, M \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \\ &= (\overline{u_1}, \overline{u_2}) \begin{pmatrix} av_1 + iv_2 \\ -iv_1 + bv_2 \end{pmatrix} \\ &= av_1\overline{u_1} + iv_2\overline{u_1} - iv_1\overline{u_2} + bv_2\overline{u_2} \end{aligned}$$

Finally,

$$\begin{aligned} \overline{\langle v, u \rangle} &= \overline{av_1\overline{u_1} + iv_2\overline{u_1} - iv_1\overline{u_2} + bv_2\overline{u_2}} \\ &= au_1\overline{v_1} - iu_1\overline{v_2} + iu_2\overline{v_1} + bu_2\overline{v_2} \\ &= \langle u, v \rangle \end{aligned}$$