Math 4355 – Spring 2021   Name:

Quiz #4

Consider the inner product space $V = L^2([0, 1])$.

(1) [8 Pts] Compute the orthogonal projection of the function $f(x) = \sin(2\pi x)$, for $x \in [0, 1]$, onto the subspace V_0 of V defined by $V_0 = \text{span}\{\phi, \psi\}$, where

$$\phi(x) = \begin{cases} 1 & 0 \le x < 1 \\ 0 & \text{otherwise} \end{cases} \qquad \psi(x) = \begin{cases} 1 & 0 \le x < \frac{1}{2} \\ -1 & \frac{1}{2} \le x < 1 \\ 0 & \text{otherwise} \end{cases}$$

(2) [3 Pts] Let $V_1 = \text{span}\{\psi_2\} \subset V$ where $\psi_2(x) = \psi(2x)$. Show that $V_1 \perp V_0$, that is, V_1 is orthogonal to V_0.

[Hint: plot ψ_2; this will help you to guide your calculation showing that ψ_2 is orthogonal to both ϕ and ψ.]