Math 4355 Name: SOLUTION

Quiz # 5

Please, type or write legibly, scan, save file as LASTNAME_FIRSTNAME_Q5.pdf and email to `dlabate@math.uh.edu` or `dlabate@uh.edu`. You need to email to me no later than 11:30AM on March 3.

Consider the function

$$f(x) = \begin{cases} 1 + \frac{2}{\pi}x & \text{if } -\frac{\pi}{2} \le x < 0 \\ 1 - \frac{2}{\pi}x & \text{if } \quad 0 \le x \le \frac{\pi}{2} \\ 0 & \text{otherwise} \end{cases}$$

(1) Sketcth the function.
(2) Compute the Fourier series, valid in the interval $[-\pi, \pi]$, of $f(x)$.

IMPORTANT: Take advantage of even/odd properties to reduce calculations.

Solution.

$$F(x) = a_0 + \sum_{k=1}^{\infty} a_k \cos(kx) + b_k \sin(kx)$$

Since f is even, the terms $b_k = 0$ for all k.

$a_0 = \frac{1}{\pi} \int_0^{\pi/2} (1 - \frac{2}{\pi}x)dx = \frac{1}{\pi}(x - \frac{x^2}{\pi})\Big|_0^{\pi/2} = \frac{1}{\pi}(\frac{\pi}{2} - \frac{\pi^2}{4\pi}) = \frac{1}{2} - \frac{\pi}{4}$

$a_k = \frac{2}{\pi} \int_0^{\pi/2} (1 - \frac{2}{\pi}x)\cos(kx)dx = \frac{2}{\pi} \int_0^{\pi/2} \cos(kx)dx - \frac{4}{\pi^2} \int_0^{\pi/2} x\cos(kx)dx$

We have:

$\frac{2}{\pi} \int_0^{\pi/2} \cos(kx)dx = \frac{2}{k\pi} \sin(kx)\Big|_0^{\pi/2} = \frac{2}{k\pi} \sin(k\frac{\pi}{2})$

$$\begin{aligned} \frac{4}{\pi^2} \int_0^{\pi/2} x\cos(kx)dx &= \frac{4}{\pi^2} \frac{x}{k} \sin(kx)\Big|_0^{\pi/2} - \frac{4}{\pi^2 k} \int_0^{\pi/2} \sin(kx)dx \\ &= \frac{4}{\pi^2} \frac{x}{k} \sin(k\frac{\pi}{2}) + \frac{4}{\pi^2 k^2} \cos(kx)\Big|_0^{\pi/2} \\ &= \frac{4}{\pi^2} \frac{x}{k} \sin(k\frac{\pi}{2}) + \frac{4}{\pi^2 k^2} \cos(k\frac{\pi}{2}) - \frac{4}{\pi^2 k^2} \end{aligned}$$

Hence, for $k \ge 1$,

$a_k = \frac{2}{k\pi} \sin(k\frac{\pi}{2}) - \frac{4}{\pi^2} \frac{x}{k} \sin(k\frac{\pi}{2}) - \frac{4}{\pi^2 k^2} \cos(k\frac{\pi}{2}) + \frac{4}{\pi^2 k^2}$