Math 4355 – Spring 2021 Name: SOLUTION

Quiz #7

Please, type or write legibly, scan, save file as LASTNAME_FIRSTNAME_Q7.pdf and email to `dlabate@math.uh.edu` or `dlabate@uh.edu`. You need to email to me no later than 11:30AM on Apr 6.

(1) *Use the definition given in class to compute the Fourier transform of*

$$f(t) = \begin{cases} t & \text{if } -\pi \le t \le \pi \\ 0 & \text{otherwise.} \end{cases}$$

SOLUTION:

Since f is an odd function, we can simplify $e^{-i\omega t} = cos(\omega t) - i\sin(\omega t)$ by dropping the cosine term. Hence we have:

$$\begin{aligned}
\hat{f}(\omega) &= \frac{1}{\sqrt{2\pi}} \int_{-\pi}^{\pi} t\, e^{-i\omega t}\, dt \\
&= -i\sqrt{\frac{2}{\pi}} \int_0^{\pi} t\, \sin(\omega t)\, dt \\
&= -i\sqrt{\frac{2}{\pi}} \left(-\frac{t}{\omega} \cos(\omega t)\Big|_0^{\pi} + \frac{1}{\omega} \int_0^{\pi} \cos(\omega t)\, dt \right) \\
&= -i\sqrt{\frac{2}{\pi}} \left(-\frac{\pi}{\omega} \cos(\omega\pi) + \frac{1}{\omega^2} \sin(\omega\pi) \right) \\
&= i\sqrt{\frac{2}{\pi}} \left(\frac{\pi\omega\cos(\omega\pi) - \sin(\omega\pi)}{\omega^2} \right)
\end{aligned}$$

NOTE: The Fourier transform of f can also be computed by applying the 'product property' to the step function:

$$g(t) = \begin{cases} 1 & \text{if } -\pi \le t \le \pi \\ 0 & \text{otherwise.} \end{cases}$$

whose Fourier transform we computed in class. Hence

$$\hat{f}(\omega) = i\frac{d}{d\omega}\left(\hat{g}(\omega)\right) = i\frac{d}{d\omega}\left(\sqrt{\frac{2}{\pi}} \frac{\sin(\omega\pi)}{\omega} \right)$$