Quiz # 9

Please, type or write legibly, scan, save file as LASTNAME_FIRSTNAME_Q9.pdf and email to dlabate@math.uh.edu or dlabate@uh.edu. You need to email to me no later than 11:30AM on March 3.

Consider the following function defined in the interval $[0, 1]$:

$$f(x) = \begin{cases} -1 & \text{if } 0 \le x < 1/4 \\ 2 & \text{if } 1/4 \le x < 1/2 \\ 4 & \text{if } 1/2 \le x < 3/4 \\ 1 & \text{if } 3/4 \le x < 1 \end{cases}$$

(1) Find the Haar wavelet decomposition of f. That is, (1a) express f in terms of the basis for V_2 and then (1b) decompose f into its component parts for W_1, W_0, V_0.
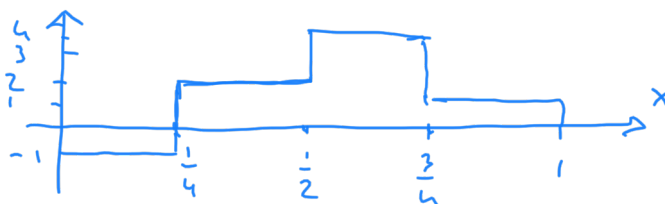
Recall that V_j and W_j are the spaces generated by $\phi(2^j x - k)$ and $\psi(2^j x - k)$, $j \ge 0$, respectively, where ϕ is the Haar scaling function and ψ is the Haar wavelet.

(2) Sketch each of the components of the Haar wavelet decomposition.

Solution.

(1a)
$$f(x) = -\phi(4x) + 2\phi(4x - 1) + 4\phi(4x - 2) + \phi(4x - 3) \in V_2$$

(1b) Haar wavelet decomposition:

$a^2 = (-1, 2, 4, 1) \to a^1 = (1/2, 5/2)$, $b^1 = (-3/2, 3/2)$,

$a^1 = (1/2, 5/2) \to a^0 = (3/2)$, $b^0 = $ -1 (-1)

Hence

$$f(x) = 3/2\phi(x) - \psi(x) - 3/2\psi(2x) + 3/2\psi(2x - 1)$$

$$\underbrace{}_{f_0} \quad \underbrace{}_{w_0} \quad \underbrace{}_{w_1}$$

where

- $f_0(x) = 3/2\phi(x) \in V_0$

- $w_0(x) = -\psi(x) \in W_0$

- $w_1(x) = -3/2\psi(2x) + 3/2\psi(2x - 1) \in W_1$