

HW #2

Please, write clearly and justify all your steps, to get proper credit for your work.

(1-3) Solve problems Ex. 9,10,11 p.35, from the textbook.

(4) Let $V = L^2([-π, π])$ and consider the subspace $V_0 \subset V$ given by

$$V_0 = \text{span} \{1, \cos x, \sin x\}$$

- (i) Find an ON basis for V_0 (note that V_0 is a subspace of $L^2([-π, π])$ so that the functions are only defined on $[-π, π]$).
- (ii) Show that the space $V_1 = \text{span} \{\cos(2x), \sin(2x)\} \subset V$ is orthogonal to V_0 . Is V_1 the orthogonal complement of V_0 ? Justify your answer.
- (iii) Find the orthogonal projection of $f(x) = \cos(3x)$ onto V_0
- (iv) Find the orthogonal projection of $f(x) = x$, for the interval $[-π, π]$, onto V_0 and onto V_1 .

(5) (Matlab project) Define a variable with the command "x=0:0.001:1;" in Matlab. This defines a row vector with values ranging from 0 to 1 in steps of 0.001 increment. Now plot the several functions $f_n(x)$ in the sequences defined in Ex.7 for $n=2$, $n=10$, and $n=50$. Think about a way to produce the piecewise definition of the functions. Refer to Matlab Help if needed. You may want to call the resulting vectors of function values f_2 , f_{10} , and f_{50} . Plotting the functions is simply done by "plot(x,f2);" and similarly for f_{10} and f_{50} . Save the plots and print them. Attach a printout of your plots to your homework, together with the Matlab code you used to generate the function values.

[NOTE: you can use Python rather than Matlab]