Math 4355 – Spring 2017 Name:

HW #3

(1) [8 Pts] Solve Ex 1, 3 and 8, p.83-84.

(2) [4 Pts] This problem is about *numerical* approximations of functions using Fourier series.

Let $f(x) = x$, for $x \in [-\pi, \pi]$. We computed in class its Fourier series, which is also a sine series (that is, the cosine terms vanish):

$$F(x) = \sum_{k=1}^{\infty} (-1)^{k+1} \frac{2}{k} \sin(kx)$$

Denote the partial sums as

$$F_N(x) = \sum_{k=1}^{\mathbb{N}} (-1)^{k+1} \frac{2}{k} \sin(kx)$$

(i) Use Matlab to produce the plots of $F_3(x)$, $F_5(x)$, $F_9(x)$. Each time, compare the plot with $f(x)$, that is, show both $F_N(x)$ and $f(x)$ on the same plot. Please, choose a window of size $[-6, 6]$, so that the behavior of the approximations at the endpoints $\pm\pi$ will be visible.

(ii) You will notice some blips in the approximating functions, near the locations $\pm\pi$. This is called the *Gibbs phenomenon.* Estimate from your graphs the height of the maximum height of the blips for $F_3(x)$, $F_5(x)$, $F_9(x)$.

NOTICE: Graphs have to be properly labelled. You are allowed to use Python rather than Matlab.